Physics of Magnetism

Chapter references are to Essentials of Paleomagnetism, UC Press, 2010
http://magician.ucsd.edu/Essentials
Magnetic units (sorry!)

- SI
- cgs

Magnetic fields as the gradient of a scalar potential

A simple dynamo

Spontaneous magnetism

- induced
- remanent
Magnetic units

- SI (mks) treatment starts with electric currents
- cgs starts with magnetic monopoles (very different!)

Why do we need to know both? Because physics uses SI and engineers use cgs. Our journals only publish in SI and (some of) our equipment is calibrated in cgs. Also, old literature uses cgs.
Let's bend the wire into a loop:

or several loops:

magnetic moment

units for m: Amperes · meters$^2$ or Am$^2$

Magnetization = \( \frac{\text{magnetic moment}}{\text{volume}} \) = A m$^{-1}$ (same as H)

Essentials: Chapter 1
Ways of thinking about magnetic fields

- Magnetic fields are vector fields, having both direction and strength.
- Strength indicated by how close the lines are together (density of magnetic flux).
- Direction indicated by directions of field lines (a.k.a. "lines of magnetic flux").

Essentials: Chapter 1
How to measure field strength?
Moving charged particles (electric currents) make magnetic fields (H) and also moving magnetic fields make electric currents

Let’s call the magnetic field that induces the current the “induction”, B

B and H are obviously similar but they do NOT HAVE THE SAME UNITS as we shall see
Could measure strength of the induction field like this:

move conductor of length $l$ at velocity $v$ through field $B$ and generate potential $V$

$$V = vlB$$

Units of $B$: $V \cdot s \cdot m^{-2}$
called a tesla (T)
But how are $B$ and $H$ related?

$$B = \mu(H + M)$$

$\mu$ is the “permeability”

in free space $M = 0$ and

$\mu = \mu_0$, the permeability of free space

units of $\mu_0$ are nasty! (see Table 1.1 in book)
Can measure direction of B with compass
turn on current and watch compass move

“Oersted experiment”
but what moves the magnet?

magnetic energy \[ E_m = - \mathbf{m} \cdot \mathbf{B} = -mB \cos \theta \]

proof from dimensional analysis:

\[ Am^2 \cdot \frac{kg}{As^2} = \text{joule} \]

handy website for fundamental units:

http://geophysics.ou.edu/solid_earth/notes/mag_basic/fundamental_units.htm
Magnetic units (sorry!)

SI

cgs - just skip this

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Spontaneous magnetism

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just as the vector of steepest descent (a vector field) is the gradient of topography (a scalar field)

the magnetic field can be thought of as the gradient of a scalar field known as the magnetic potential

\[ \mathbf{H} = -\nabla \psi_m \]

Essentials: Appendix A
Relation of magnetic potential to the field

\[ \psi_m = \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} = \frac{m \cos \theta}{4\pi r^2} \]

\[ H_r = -\frac{\partial \psi_m}{\partial r} = \frac{1}{4\pi} \frac{2m \cos \theta}{r^3} \]

\[ H_\theta = -\frac{1}{r} \frac{\partial \psi_m}{\partial \theta} = \frac{m \sin \theta}{4\pi r^3} \]
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simple disk dynamo

Numerical simulation

Elsasser 1958; Butler 1992

Glatzmaier & Roberts (1995)
Take home message

- Magnetism results from the motion of charged particles: electrical currents, electronic orbits and spins
Magnetic units (sorry!)

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Magnetic fields as the gradient of a scalar potential

A simple dynamo

- Spontaneous magnetism
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We just learned: “All magnetism results from currents”
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So where are the currents here?
Classical view: electron scooting around nucleus

In quantum mechanics: orbit is stabilized. Energy must be quantized. Electron must satisfy the “wave equation”
Classical view: electron scooting around nucleus

Doesn’t work because electron would eventually crash into the nucleus

In quantum mechanics: orbit is stabilized. Energy must be quantized. Electron must satisfy the “wave equation”
lowest energy solution to wave equation

Electron density plot

Essentials: Chapter 3
lowest energy solution to wave equation

called the 1s “shell”
examples of higher energy shells:

\( s \) (l,m=0)
\( p \) (l,m=1)
\( d \) (l,m=2)
\( d \) (l=2,m=0)
examples of higher energy shells:

one electron orbiting in one of these shells generates a magnetic moment

\[ m_b = \frac{\hbar q_e}{2 \mu_e} = 9.2710^{-24} \frac{\text{kg} \cdot \text{m}^2}{\text{s}} \cdot \frac{\text{C}}{\text{kg}} = 9.2710^{-24} \text{Am}^2 \]
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called one Bohr magneton

Essentials: Chapter 3

Thursday, April 7, 2011
summary of quantum numbers

l - Principal quantum number: l=0 (s shell), l=1, (p shell), l=2 (d shell), etc.

spin: \[ \pm \frac{1}{2} \] “up” or “down”

spin also generates one \[ m_b \]
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m - azimuthal quantum number: gives orbital angular momentum

spin: $\pm \frac{1}{2}$ “up” or “down”

spin also generates one $m_b$
summary of quantum numbers

l - Principal quantum number: \( l=0 \) (s shell), \( l=1 \) (p shell), \( l=2 \) (d shell), etc.

m - azimuthal quantum number: gives orbital angular momentum

n - energy level: s1, s2, s3 are larger and larger shells

spin: \( \pm \frac{1}{2} \) “up” or “down”

spin also generates one \( m_b \)
Rules for filling electronic shells
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No two electrons may have the same set of quantum numbers. Because spin can be “up” or “down”, two electrons fit in one shell.
Rules for filling electronic shells

- No two electrons may have the same set of quantum numbers. Because spin can be “up” or “down”, two electrons fit in one shell.

- Pauli’s Exclusion principle
Electrons are added so that the spins remain as parallel as possible (Hund’s Rule).
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Induced magnetization:
comes from electronic orbits
AND
electronic spins
moment from electronic orbit
moment from electronic orbit

Electronic orbit has angular momentum vector $\mathbf{L}$
moment from electronic orbit

Electronic orbit has angular momentum vector $\mathbf{L}$

$H$ exerts a torque on $\mathbf{L}$, shifting it by $\Delta L$ (Larmor precession)
moment from electronic orbit

Electronic orbit has angular momentum vector $\mathbf{L}$

$H$ exerts a torque on $\mathbf{L}$, shifting it by $\Delta L$

(Larmor precession)

precession creates moment $\Delta m$
Electronic orbit has angular momentum vector $\mathbf{L}$

$\mathbf{H}$ exerts a torque on $\mathbf{L}$, shifting it by $\Delta \mathbf{L}$

(Larmor precession)

Precession creates moment $\Delta m$

Induced magnetization $M_I$ is volume normalized moment
Orbitally induced magnetization called “diamagnetism”
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\[ M_I = \chi_d H \]

\( \chi_d \) is diamagnetic susceptibility
Orbitally induced magnetization called “diamagnetism”

\[ M_I = \chi_d H \]

\( \chi_d \) is diamagnetic susceptibility

- \( \chi_d \) is negative
- \( \chi_d \) is temperature independent
Spin induced magnetization called “paramagnetism”

\[ M_I = \chi_p H \]

\( \chi_p \) is “paramagnetic susceptibility”
rules

- Each unpaired spin contributes a dipole moment (one Bohr magneton)
- In the absence of an applied field, the moments are essentially random
- An applied field acts to align the spins
- There is a competition between thermal energy ($kT$) and magnetic energy ($mB \cos \theta$)
Probability density of a given electron to have magnetic energy $E_m$

$$P(E) \propto \exp(-E_m/kT)$$
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$$P(E) \propto \exp(-E_m/kT)$$

from this, magnetic energy, which is the degree of alignment with the field, depends on B and T

$$a = mB/kT$$
Probability density of a given electron to have magnetic energy $E_m$

$$P(E) \propto \exp\left(-\frac{E_m}{kT}\right)$$

from this, magnetic energy, which is the degree of alignment with the field, depends on $B$ and $T$

$$a = \frac{mb}{kT}$$

$$\frac{M}{M_s} = \left[\coth a - \frac{1}{a}\right] = \mathcal{L}(a)$$

(see Appendix A.2.1 for derivation)
\[
\frac{m}{kT}B = \chi_p B
\]

is positive and has a strong temperature dependence
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Magnetic remanence:
the magnetization that stays when
the field turns off
a.k.a “spontaneous magnetization”

Stems from cooperative behavior between
neighboring spins – minimizes “exchange
energy”
Types of spin alignment

ferronmagnetic
Types of spin alignment

ferrōmagnetic

A sites
tetrahedral
Fe$^{3+}$

B sites
octahedral
Fe$^{3+}$, Fe$^{2+}$

Essentials: Chapter 3
Types of spin alignment

ferromagnetic  ferrimagnetic

Essentials: Chapter 3

A sites
tetrahedral
Fe$^{3+}$

B sites
octahedral
Fe$^{3+}$, Fe$^{2+}$

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Types of spin alignment

- Ferromagnetic
- Ferrimagnetic
- Anti-ferromagnetic

Essentials: Chapter 3
Types of spin alignment

- **Ferromagnetic**
- **Ferrimagnetic**
- **Antiferromagnetic**
- **Spin canted**

Essentials: Chapter 3
Types of spin alignment

- **Ferromagnetic**
  - Tetrahedral Fe$^{3+}$
  - Octahedral Fe$^{3+}$, Fe$^{2+}$

- **Ferrimagnetic**

- **Anti-ferromagnetic**

- **Spin canted anti-ferromagnetic**

- **Defect**

Essentials: Chapter 3

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When all spins are perfectly parallel, magnetization is at saturation \( M = M_s \)
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as \( T \) goes up, crystals expand and exchange energy becomes weaker - alignment \((M)\) goes down

\[
\frac{M}{M_s} < 1
\]
When all spins are perfectly parallel, magnetization is at saturation \( M = M_s \)

as \( T \) goes up, crystals expand and exchange energy becomes weaker - alignment (M) goes down

\[
\frac{M}{M_s} < 1
\]

\( T > T_c \)

no exchange is zero and \( M \) is paramagnetic

Essentials: Chapter 3
Take home message

- Magnetism results from the motion of charged particles: electrical currents, electronic orbits and spins
- Ferro magnetism is the results of cooperation in crystals between neighboring spins