

# Lecture 4

- Things we might want to know about the geomagnetic field
- Components of the geomagnetic field
- The geomagnetic reference field
- All sorts of magnetic poles
- observables

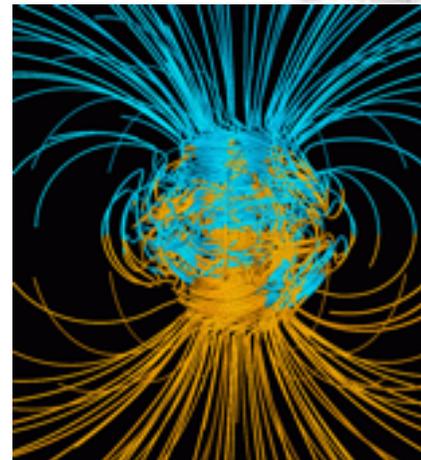
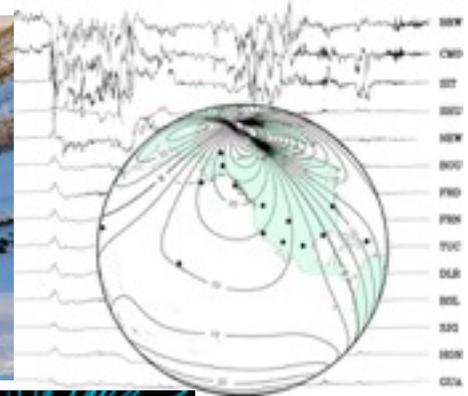
# Things we might want to know

- Where does the field come from?
- How long has it existed?
- What is its average strength and how does it vary?
- What can it tell us about processes in the deep Earth?



# How can we study the magnetic field?

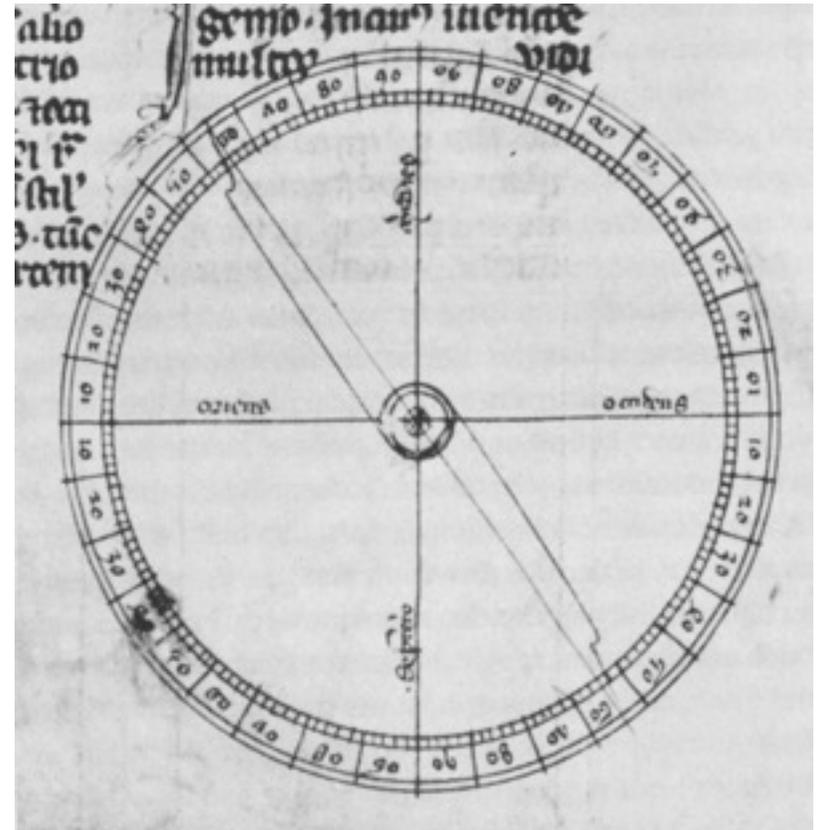
- Direct observations (satellites, ground based observatories, other human measurements [captain's logs...])
- Numerical simulation (e.g., Glatzmaier & Roberts, 1995) and laboratory dynamos (e.g., Aurnou and friends)
- Indirect records (archaeological and geological)



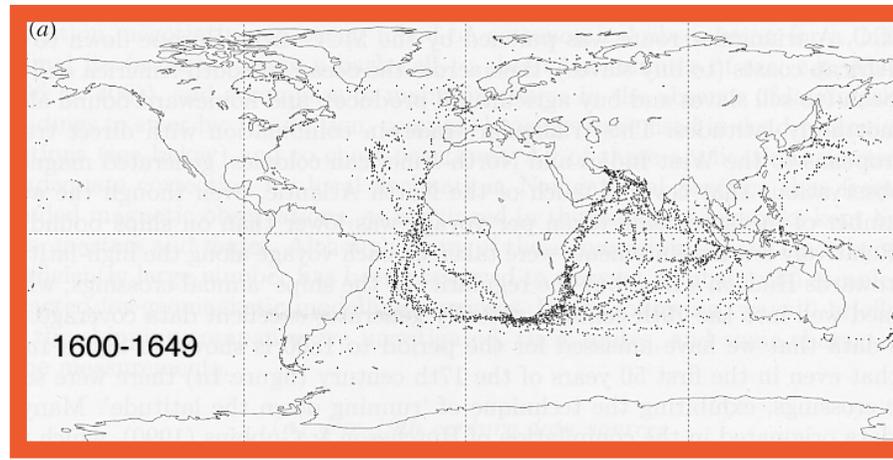
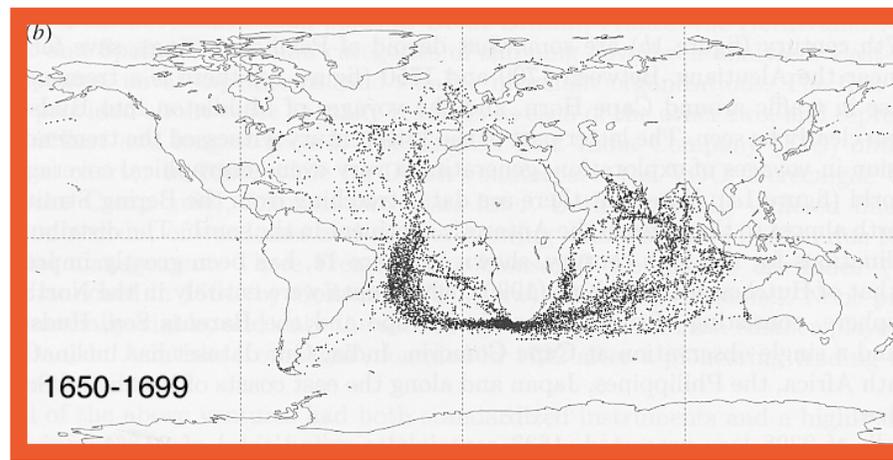
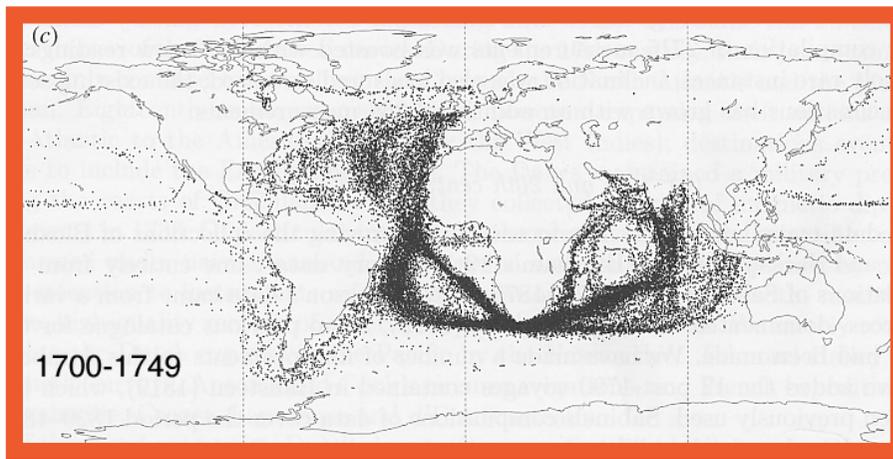
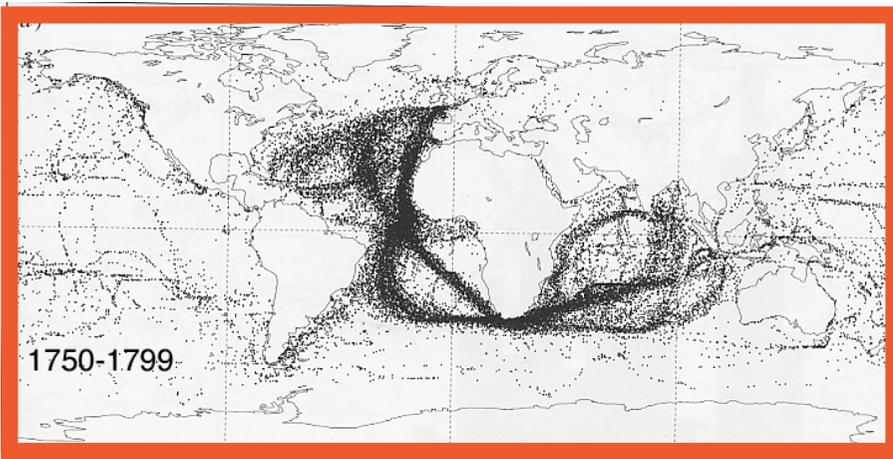
1st century CE????



Perigrinus 1269 CE



Direct observations go back a long way!



Gilbert (1544-1603)

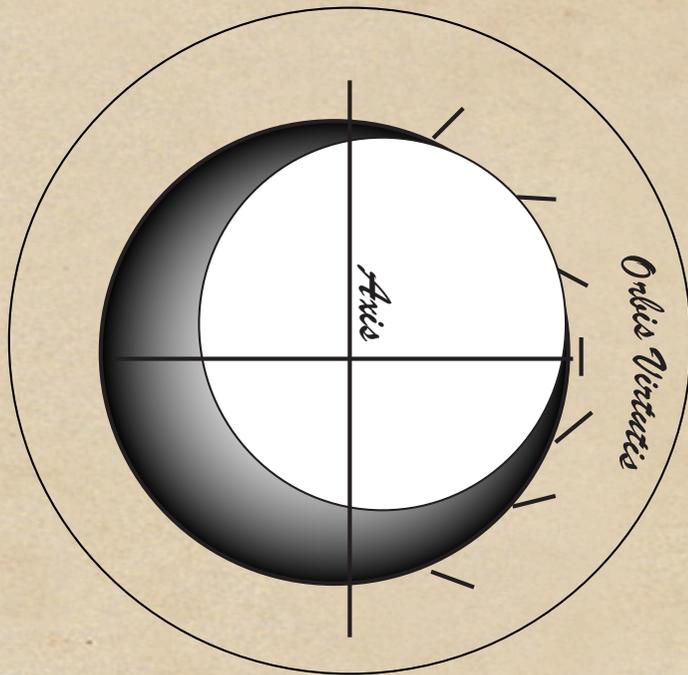
Physician to Queen  
Elizabeth

Studied magnetism of  
the earth and magnetic  
materials

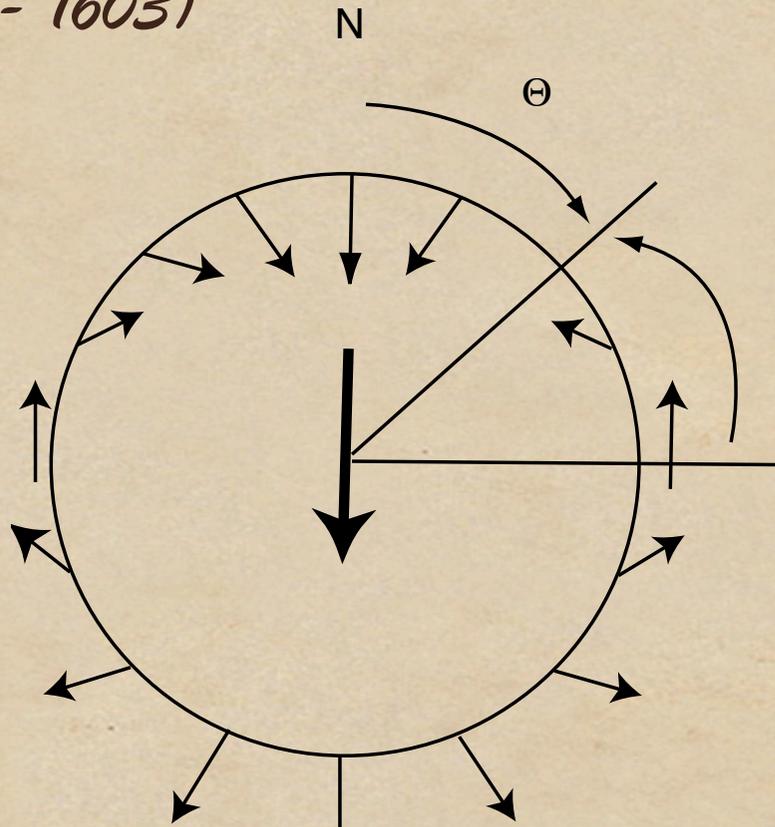


# Studies of a lodestone ball or "terrella"

Gilbert (1544-1603)



The field of a magnetic dipole



The Earth itself is a great magnet!

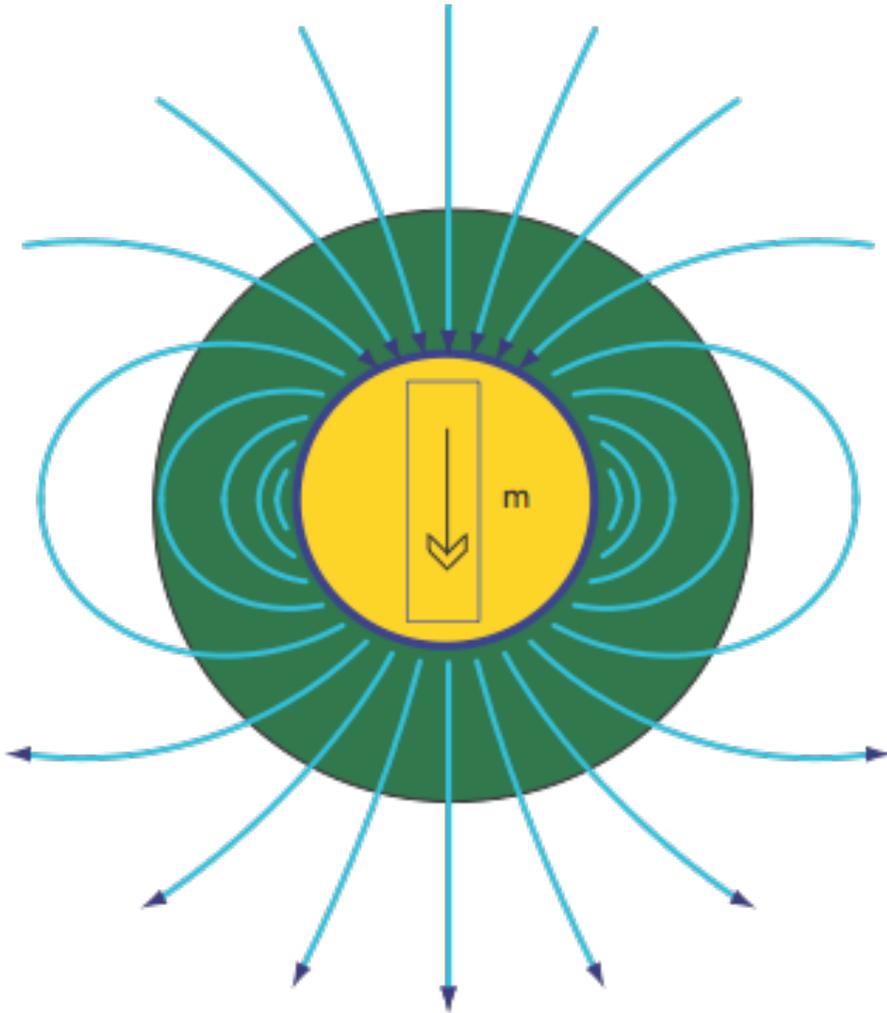
*"We may see how far from unproductive magnetick philosophy is, how agreeable, how helpful, how divine! Sailors when tossed about on the waves with continuous cloudy weather, and unable by means of the celestial luminaries to learn anything about the place or the region in which they are, with a very slight effort and with a small instrument are comforted, and learn the latitude of the place."*

*Gilbert 1600*

# Uses of the "bar magnet" hypothesis

- Sailors can "learn their latitudes" (not very accurately) by measuring dip of the field.
- Geologists can "learn the latitude" of rocks at their birth and track the movements of the continents through time.
- example is: shallow dip in sea-level glacial deposits used to support "snow ball earth" hypothesis.

## Geocentric Axial Dipole (GAD); $D=0$



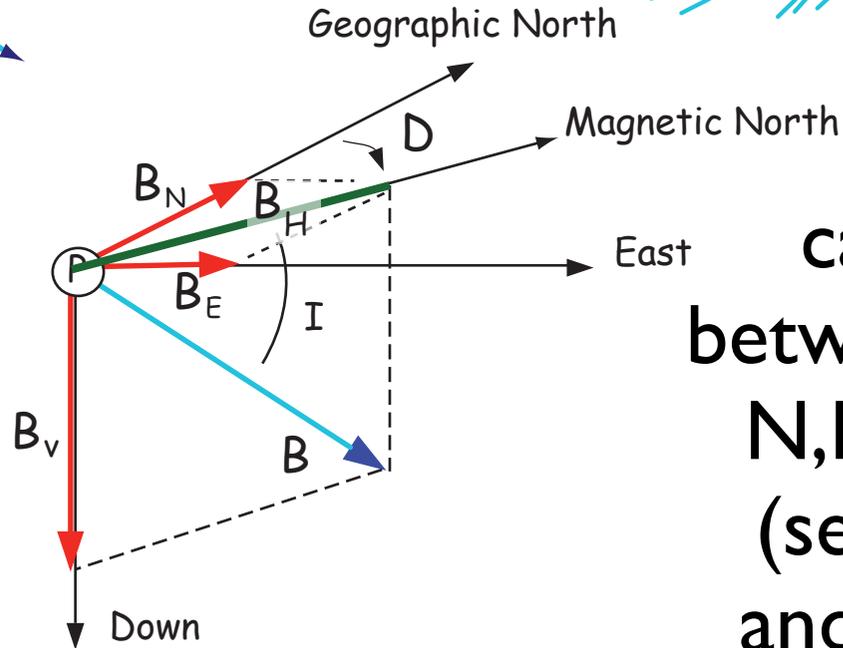
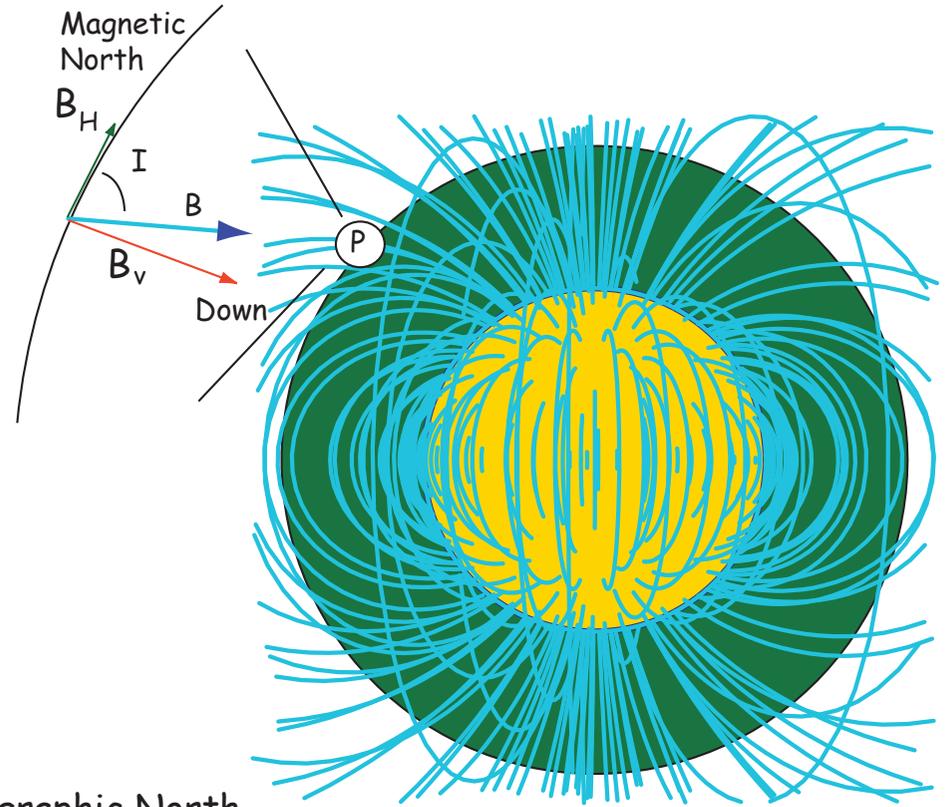
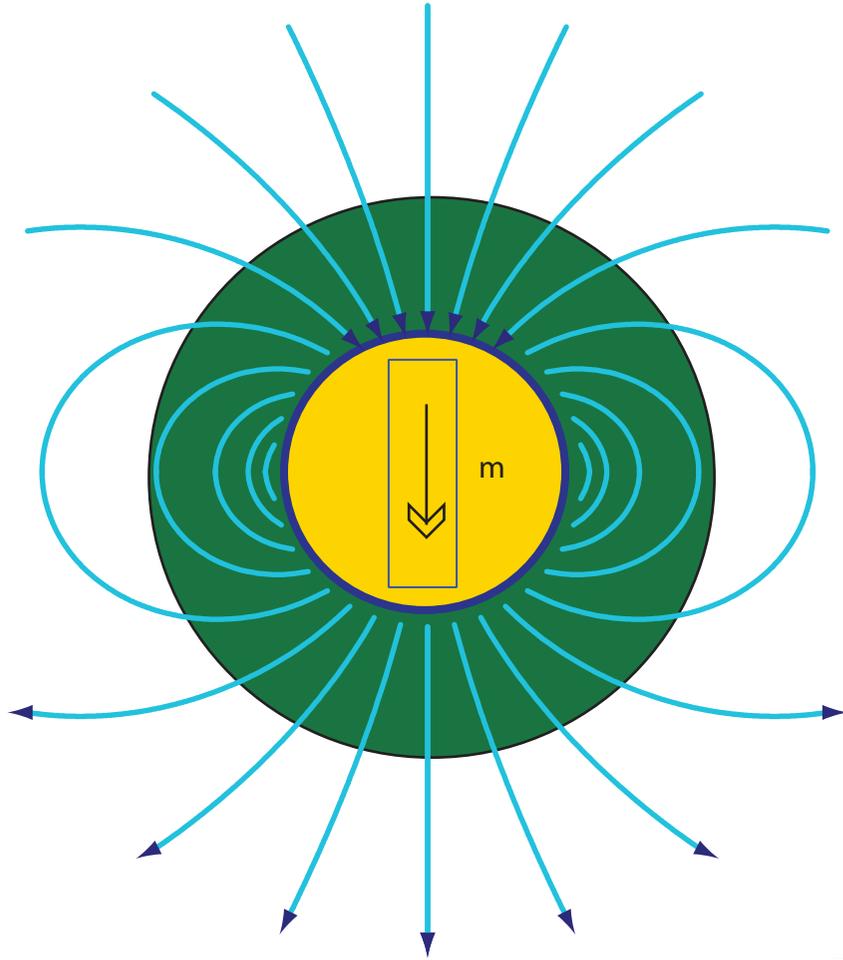
The intensity of the field is twice as strong in the polar than in the equator

The inclination varies as a function of latitude from 0- at equator to vertical at poles

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# But the field is more complicated than that



can convert  
between  $D, I, B$  and  
 $N, E, V$  with trig  
(see Lecture 3  
and Chapter 2)

Recall from the last lecture:

A magnetic field ( $\mathbf{H}$ ) is the gradient of a magnetic potential:

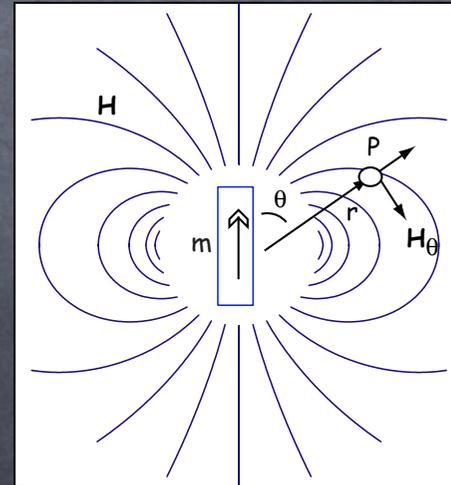
$$\mathbf{H} = -\nabla\psi_m$$

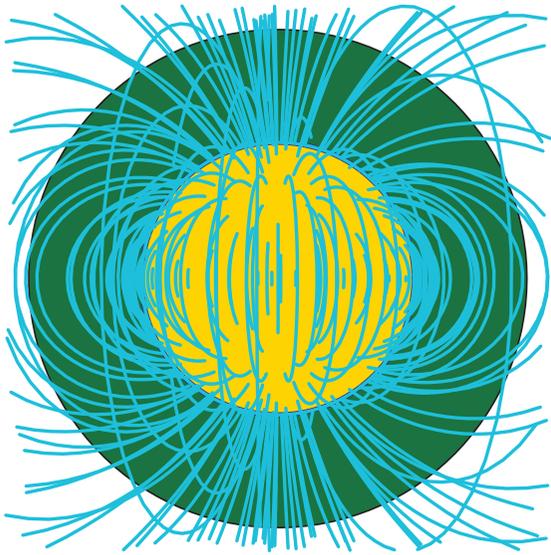
simplest case: 
$$\psi_m = \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} = \frac{m \cos \theta}{4\pi r^2}$$

So, knowing  $m$ , we can evaluate  $\mathbf{H}$  anywhere and vice versa

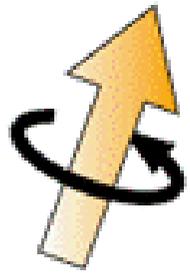
$$H_r = -\frac{\partial\psi_m}{\partial r} = \frac{1}{4\pi} \frac{2m \cos \theta}{r^3}$$

$$H_\theta = -\frac{1}{r} \frac{\partial\psi_m}{\partial \theta} = \frac{m \sin \theta}{4\pi r^3}$$

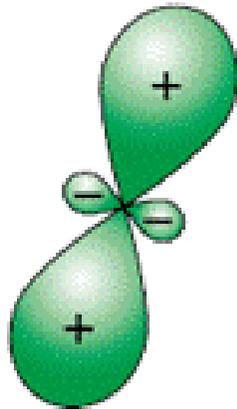




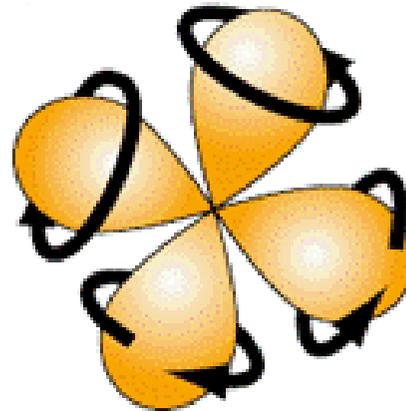
The “real” field is sum of dipole plus a lot of more complicated fields:



dipole



quadrupole



octupole

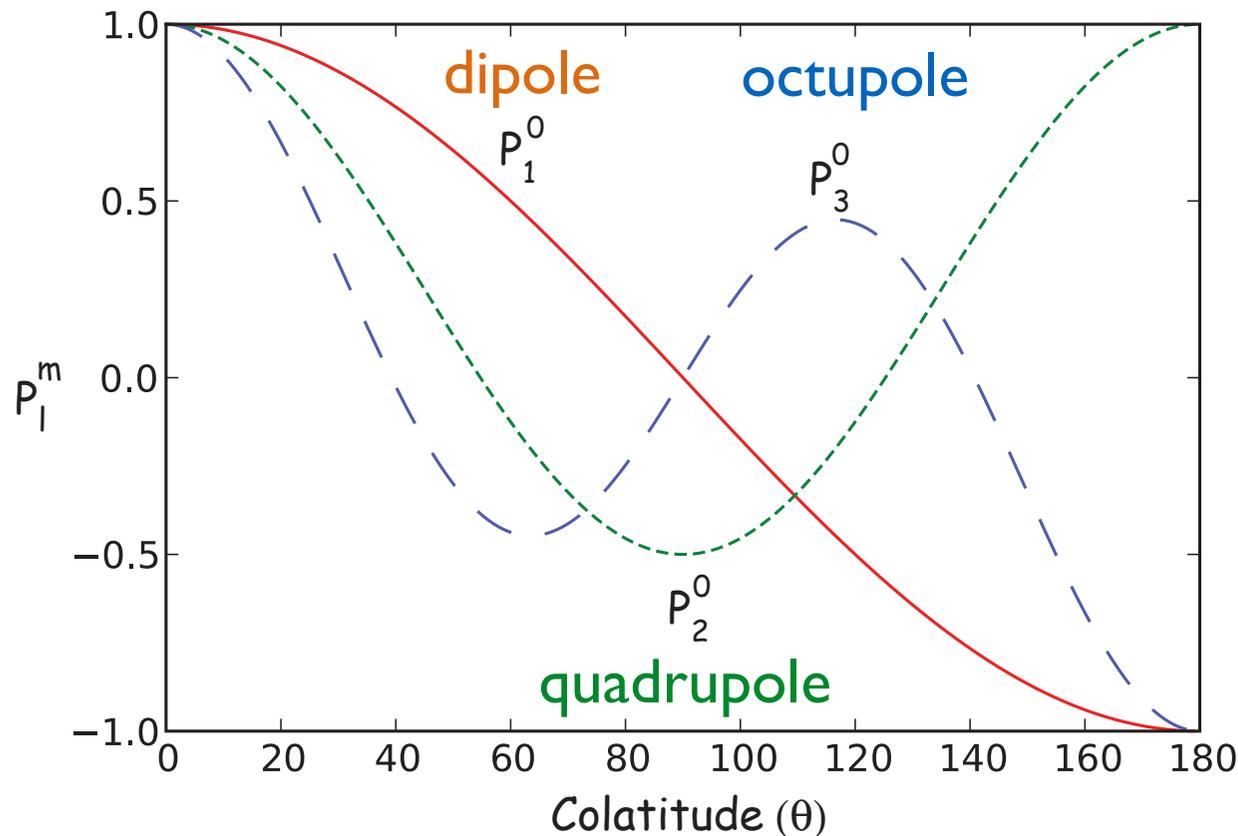
dipole potential:

$$\psi_m = \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} = \frac{m \cos \theta}{4\pi r^2}$$

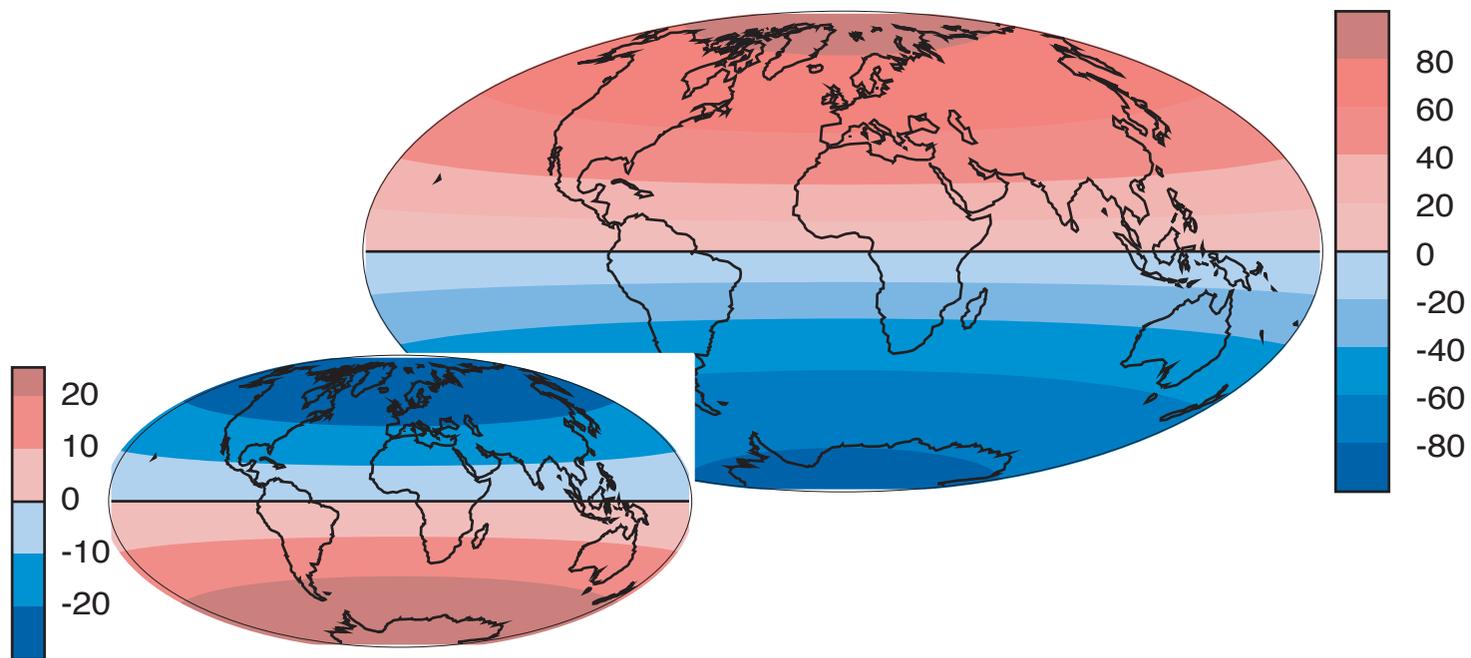
more general case:

$$\psi_m(r, \theta, \phi) = \frac{a}{\mu_o} \sum_{l=1}^{\infty} \sum_{m=0}^l \left(\frac{a}{r}\right)^{l+1} P_l^m(\cos \theta) (g_l^m \cos m\phi + h_l^m \sin m\phi)$$

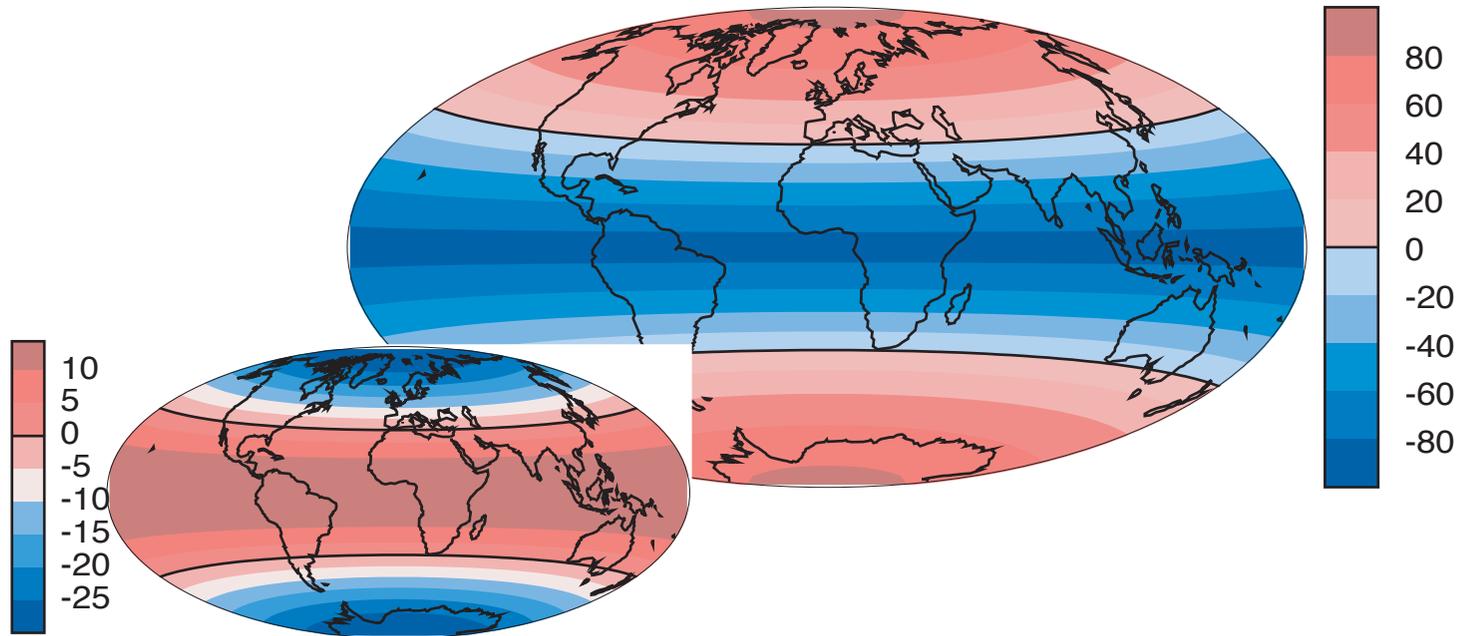
$$P_1^0 = \cos \theta, P_2^0 = \frac{1}{2}(3 \cos^2 \theta - 1), \text{ and } P_3^0 = \frac{1}{2} \cos \theta (5 \cos^3 \theta - 3 \cos \theta)$$



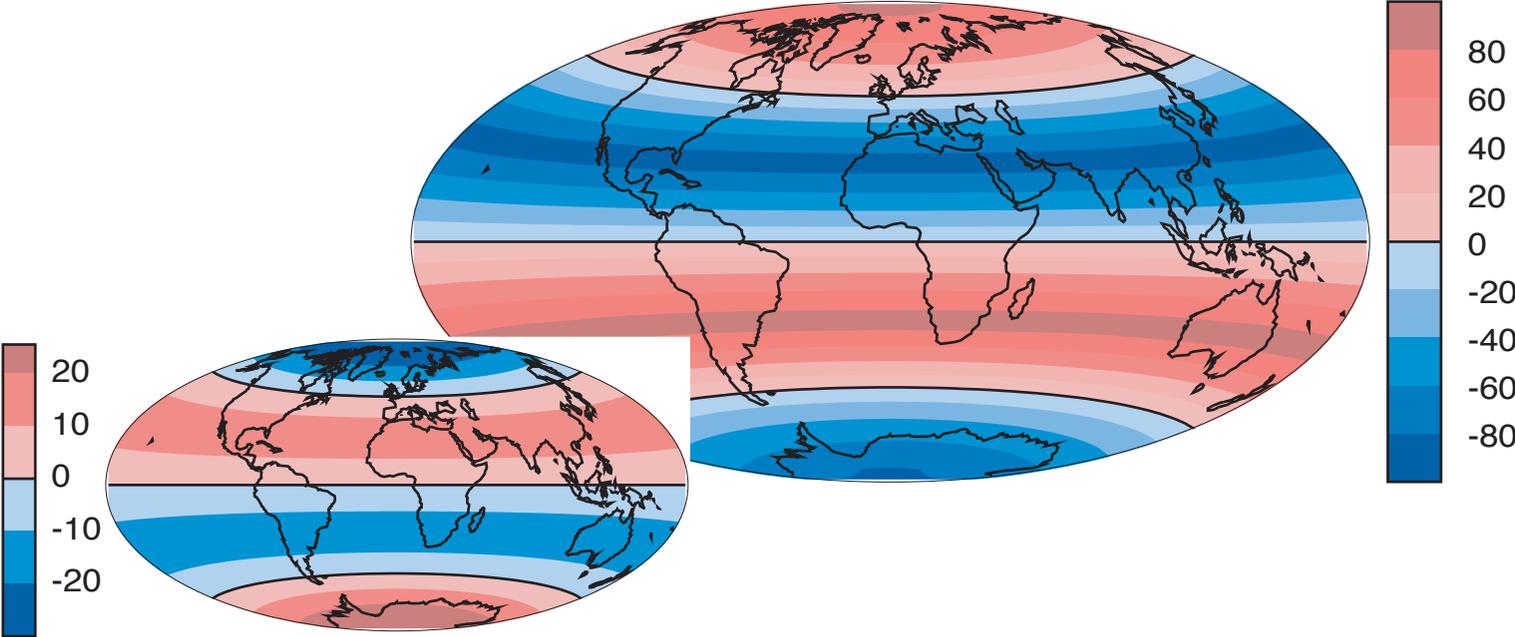
# Dipolar field and potential



# Quadrupolar field and potential



# Octupolar field and potential



## Geomagnetic potential:

$$\psi_m(r, \theta, \phi) = \frac{a}{\mu_o} \sum_{l=1}^{\infty} \sum_{m=0}^l \left(\frac{a}{r}\right)^{l+1} P_l^m(\cos \theta) (g_l^m \cos m\phi + h_l^m \sin m\phi)$$

So, we need a whole list of numbers  $g_l^m$  and  $h_l^m$

(International Geomagnetic Reference Fields - IGRF)

These are best-fits from the observations for a given year.

These lists allow us to predict the geomagnetic field vector at any point outside the core

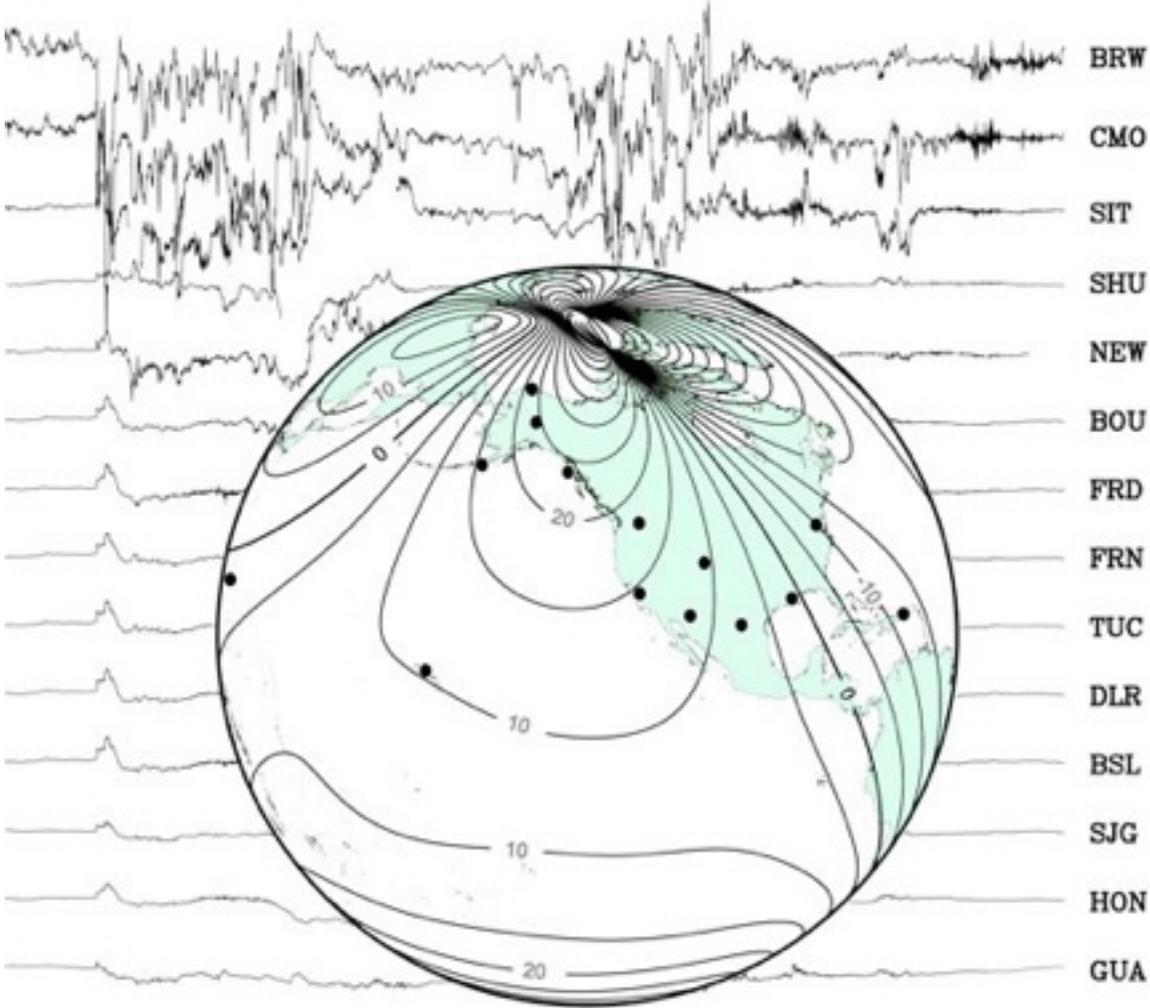
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# Getting a reference field model

- Make a bunch of measurements of the geomagnetic field vector all over the world
- Find the best-fit coefficients to a special equation (the geomagnetic potential equation)
- These coefficients are the international geomagnetic reference field
- Can calculate the geomagnetic field vector ANYWHERE outside the source (the core)

# geomagnetic observatories and satellites



Observatories since 1800s; IGRFs since 1900

TABLE 2.1: IGRF, 10TH GENERATION (2005) TO  $l = 6$ .

$l$	$m$	$g$ (nT)	$h$ (nT)	$l$	$m$	$g$ (nT)	$h$ (nT)
1	0	-29556.8	0	5	0	-227.6	0
1	1	-1671.8	5080	5	1	354.4	42.7
2	0	-2340.5	0	5	2	208.8	179.8
2	1	3047	-2594.9	5	3	-136.6	-123
2	2	1656.9	-516.7	5	4	-168.3	-19.5
3	0	1335.7	0	5	5	-14.1	103.6
3	1	-2305.3	-200.4	6	0	72.9	0
3	2	1246.8	269.3	6	1	69.6	-20.2
3	3	674.4	-524.5	6	2	76.6	54.7
4	0	919.8	0	6	3	-151.1	63.7
4	1	798.2	281.4	6	4	-15	-63.4
4	2	211.5	-225.8	6	5	14.7	0
4	3	-379.5	145.7	6	6	-86.4	50.3
4	4	100.2	-304.7				

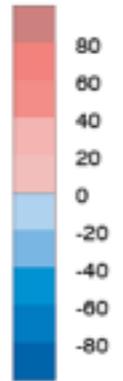
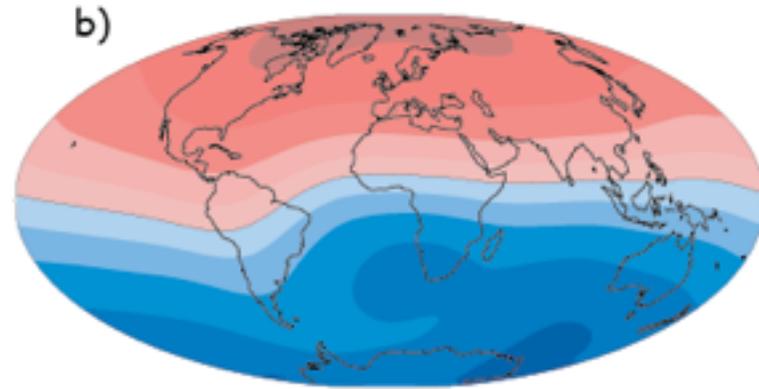
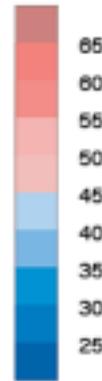
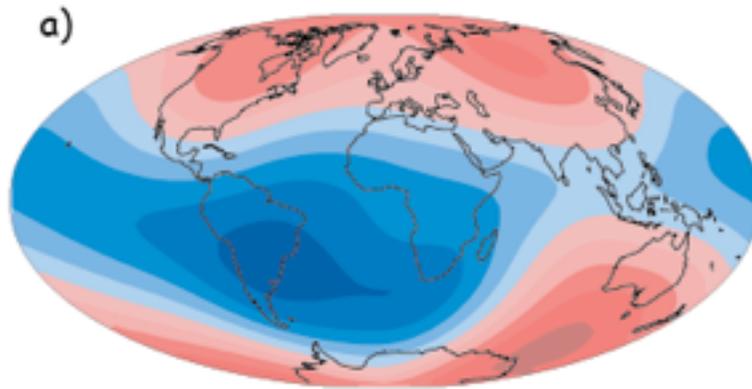
current version is IGRF-12

Thebault et al. (2015): [dx.doi.org/10.1186/s40623-015-0228-9](https://doi.org/10.1186/s40623-015-0228-9)

online calculator: <http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html>

B

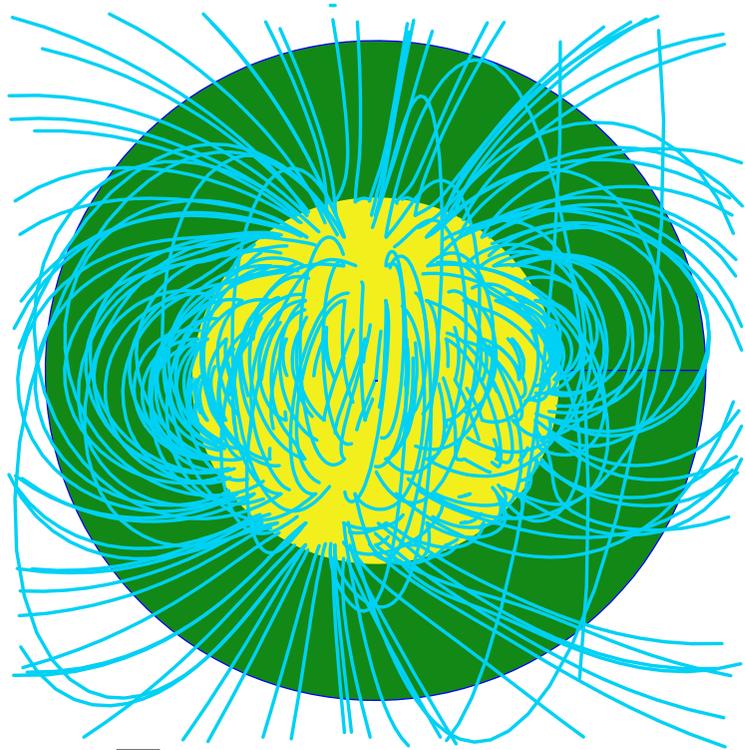
Inc



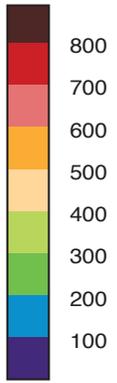
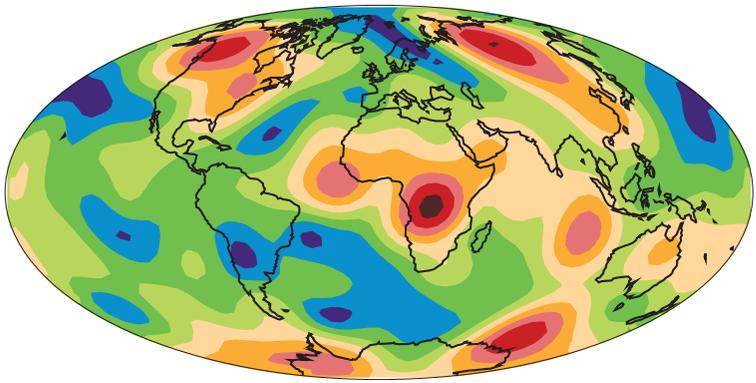
Still has increase in B towards the pole and change in inc with latitude, but more complicated

90% dipolar though!

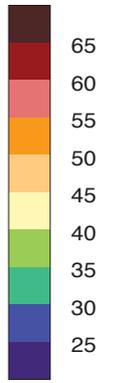
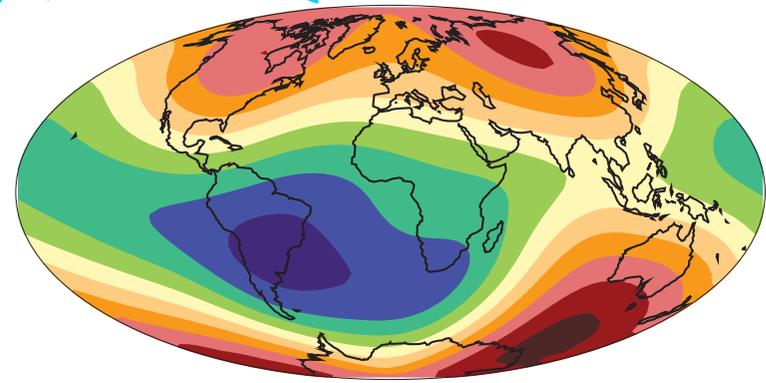
Different projections  
of the same thing -  
IGRF 2005



Magnetic  
field lines



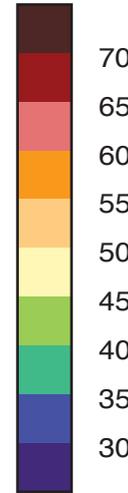
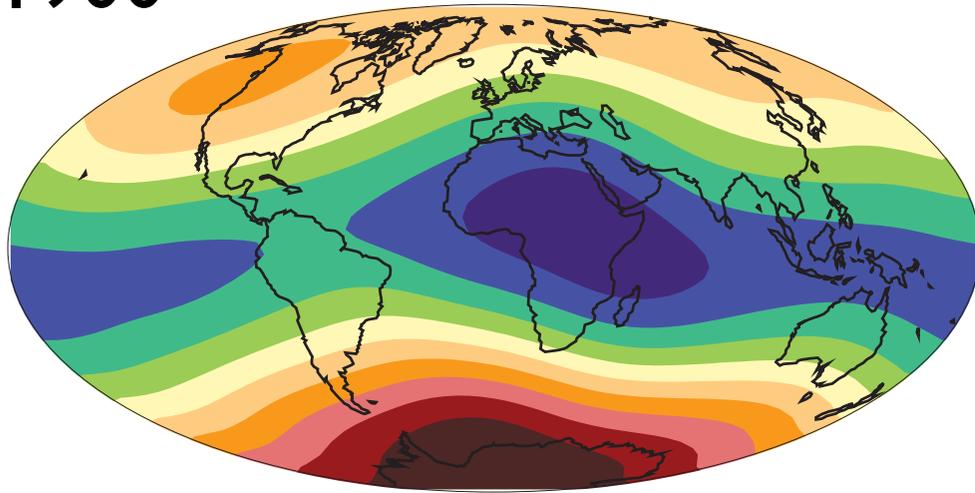
strength at the CMB



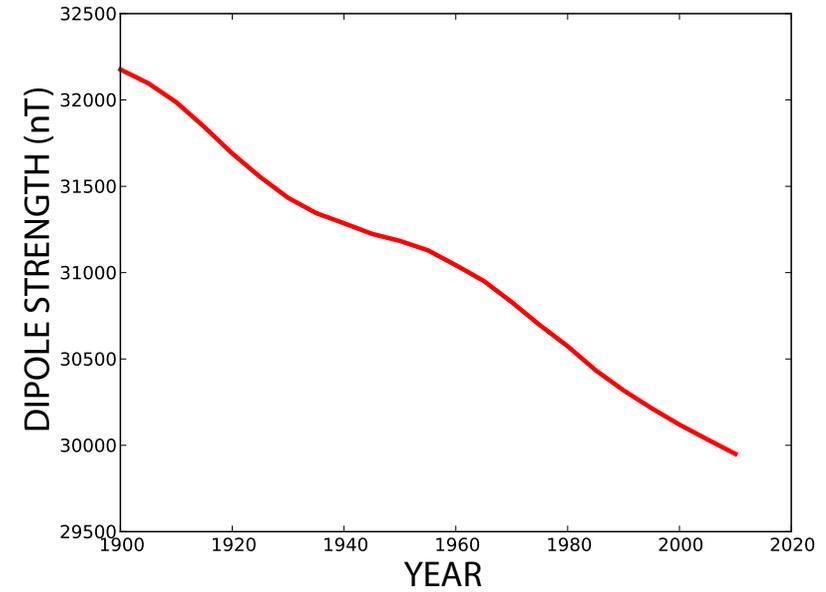
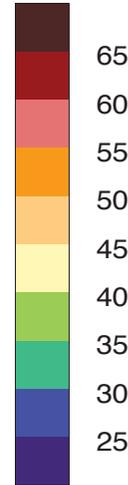
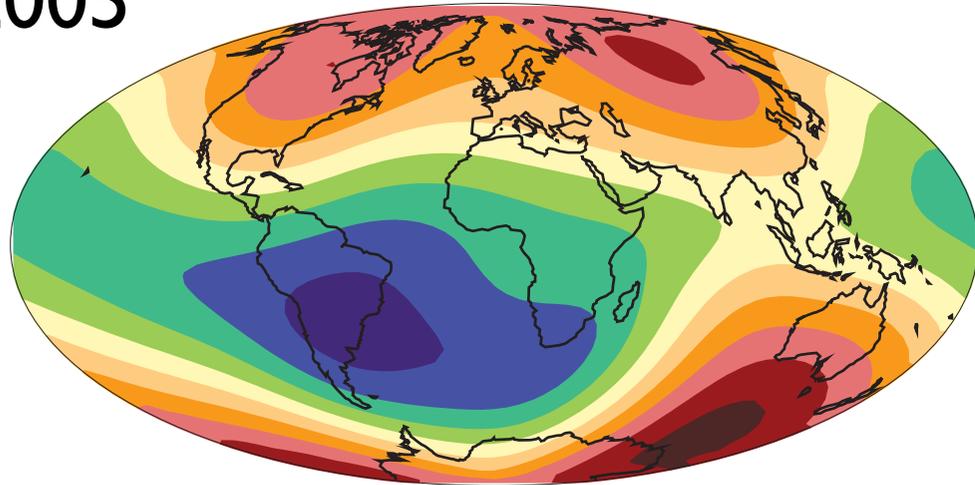
strength at the surface

# Secular variation

1900



2005

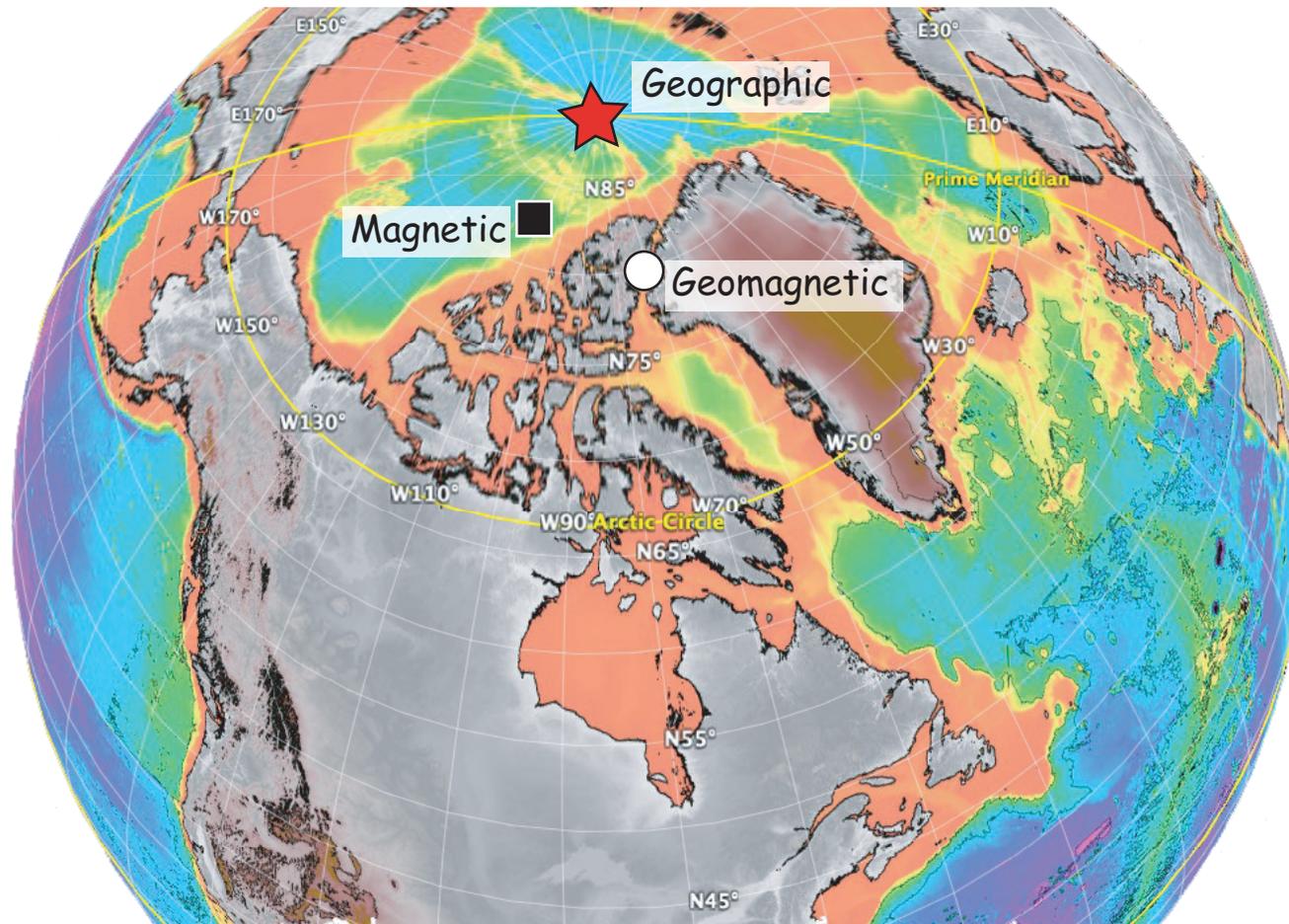


8% drop over  
last century

# Lecture 4

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# Our first three poles (more to come)



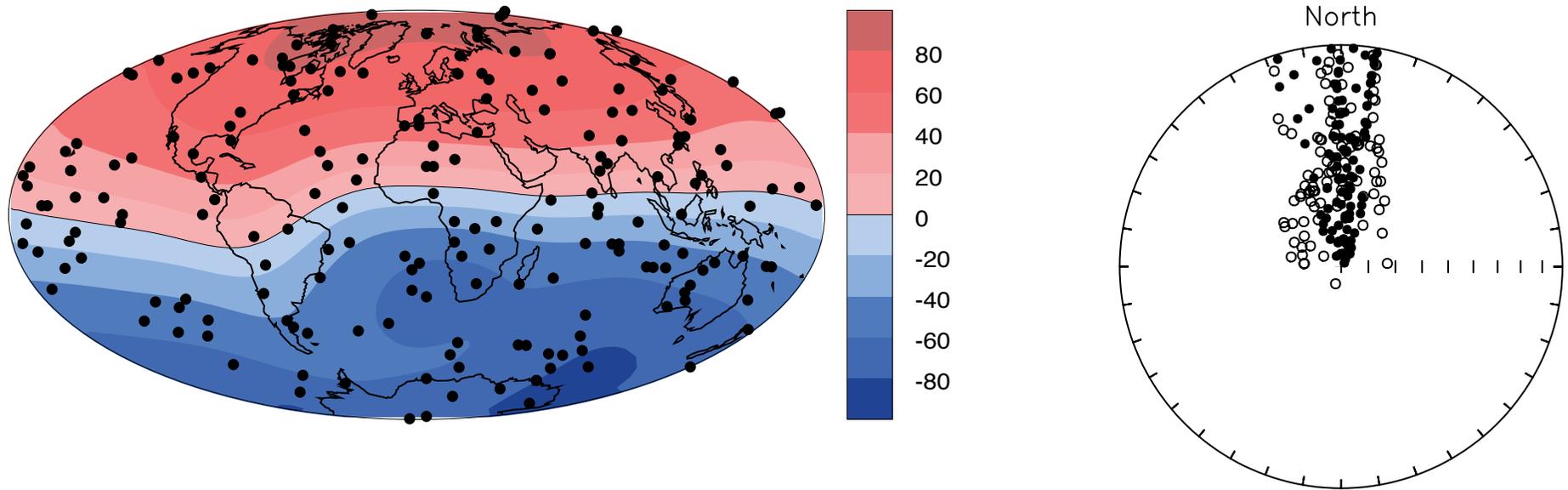
geographic: spin axis (the GAD pokes out here)

magnetic: places where field is vertical

geomagnetic: piercing points of best-fit dipole

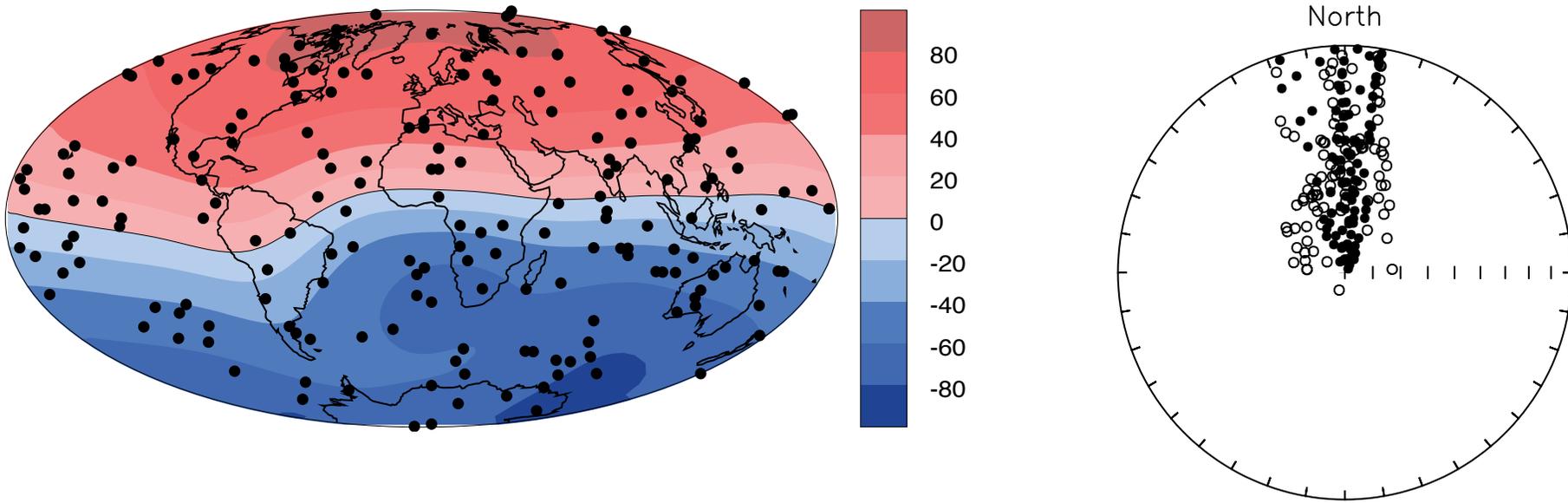
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- **observables**

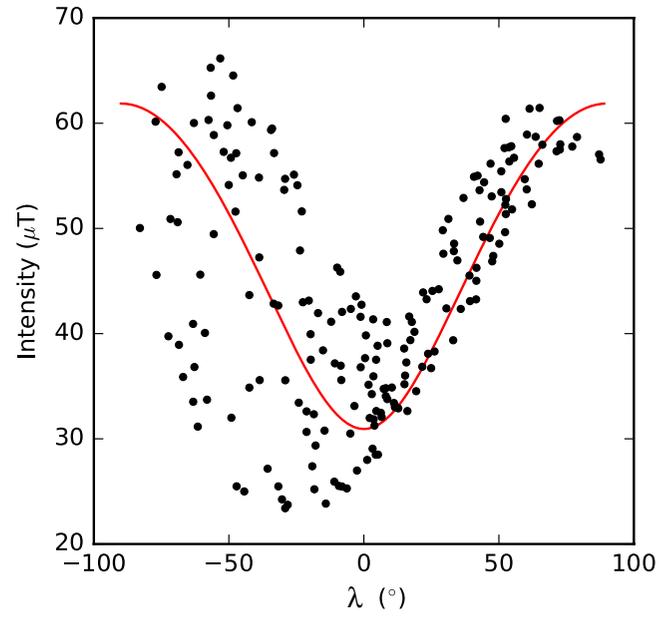
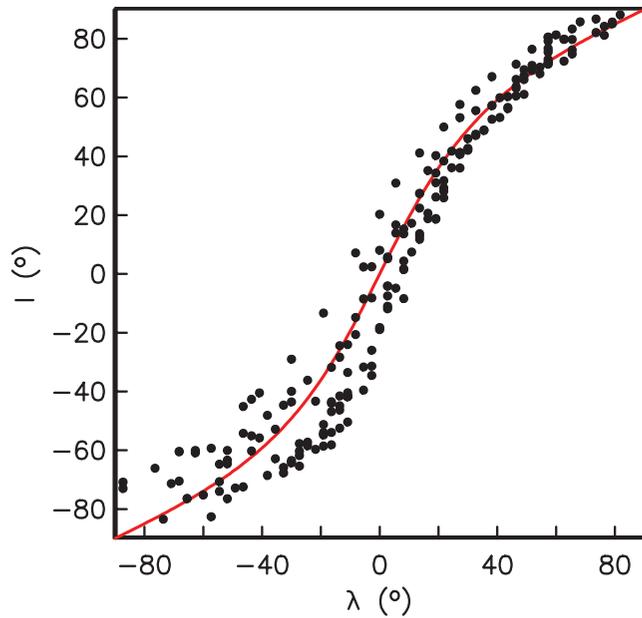


Declination and inclination observed around the globe  
and plotted on equal area projection

You will do something like this for  
problems 2.1 & 2.2 in your assignment



evaluate the field vector at these points



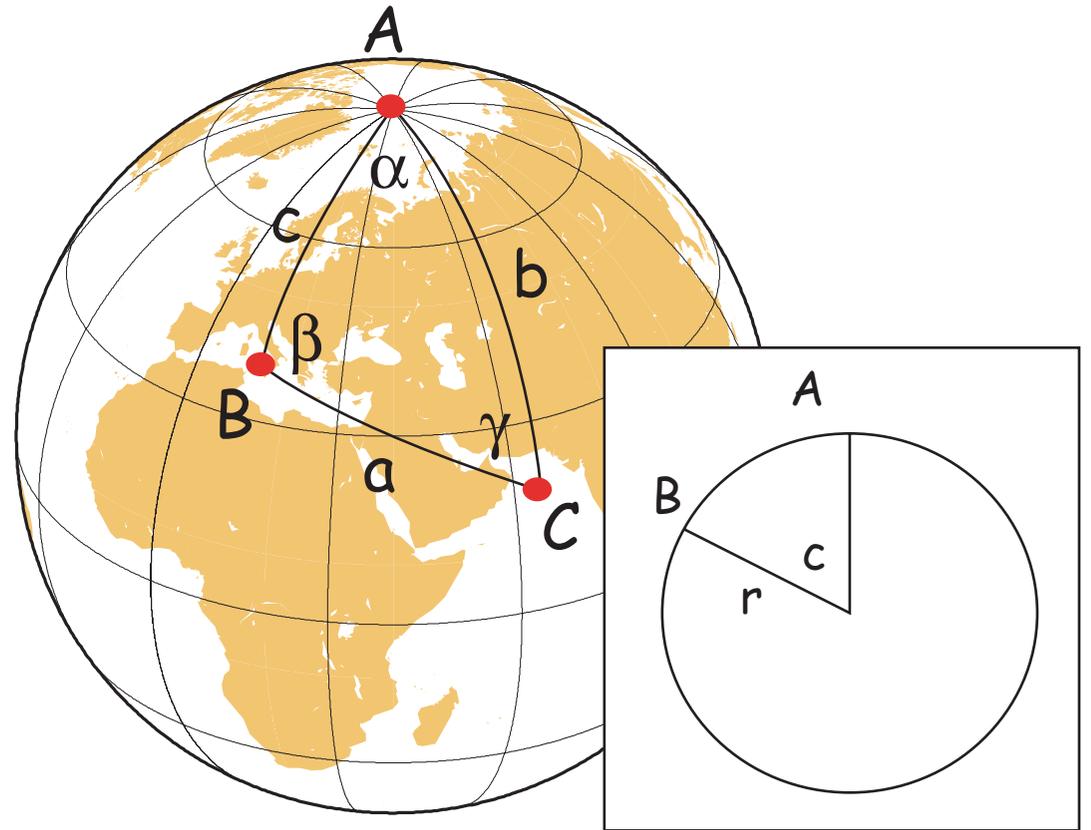
# Lecture 4

- Things we might want to know about the geomagnetic field
- Components of the geomagnetic field
- The geomagnetic reference field
- All sorts of magnetic poles: 2 more!
- observables

# But first: a math refresher spherical trigonometry

law of sines:

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}$$



law of cosines:

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$$



# The last pole!

- When you average a bunch of poles together (to average out secular variation), you get one more pole
- a Paleomagnetic Pole. This is assumed to be equivalent to GAD: aligned with the spin axis and centered in the earth.
- Can you use these poles to calculate paleolatitude, e.g.

# Assignment

- Make sure you have properly installed PmagPy (now you need it!).
- Follow the instructions in the cookbook for moving the data\_files directory to your Homework folder
- Do problems 2.1 and 2.2 in the online version of the textbook.