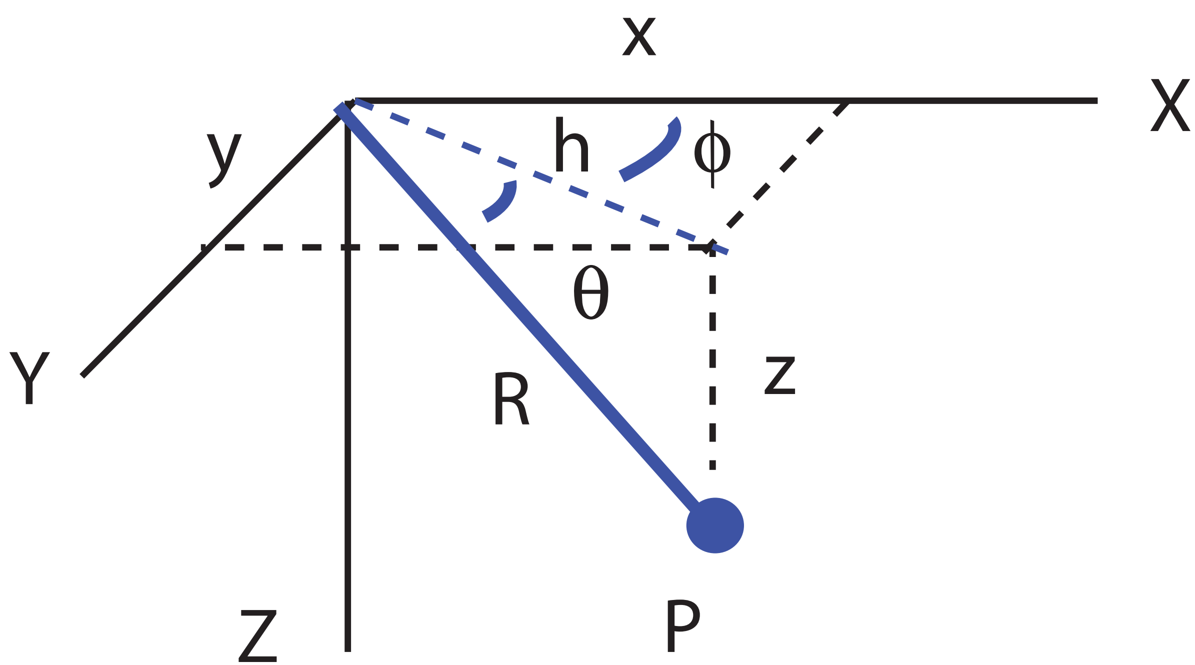
Announcements

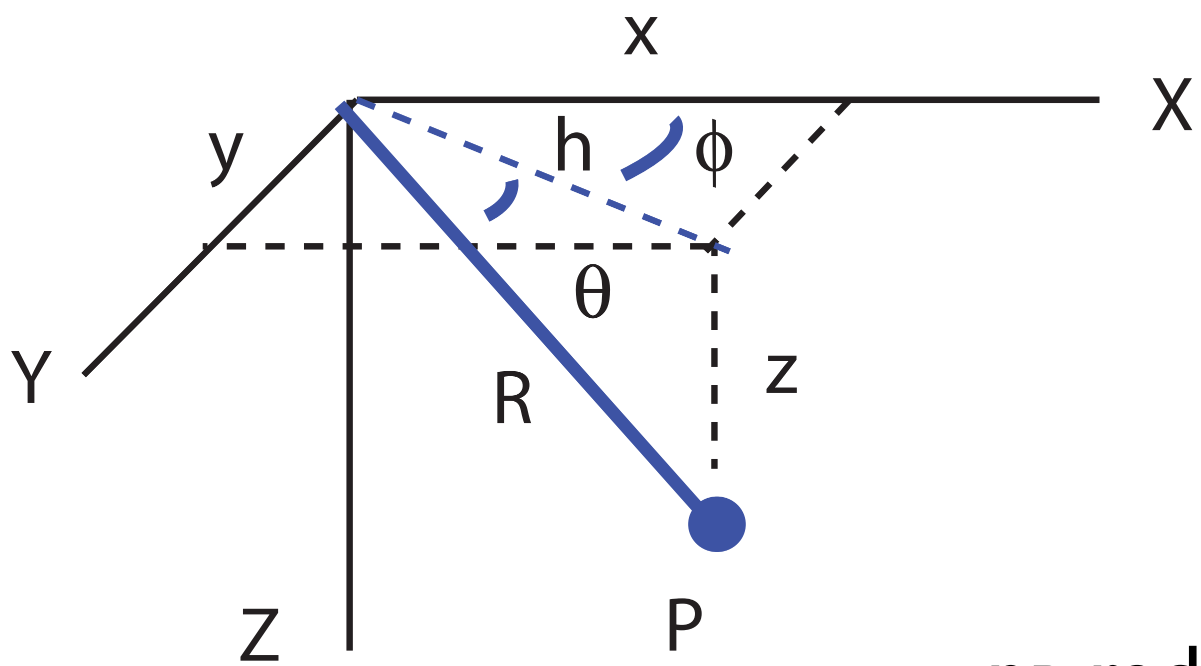
- From now on, the problem sets from each week's homework assignments will be the following Wednesday.
- Late assignments will not be accepted.
- I will post the solutions on line after class on Wednesdays.
- Start thinking about project ideas.
- Oh - and use the online version the textbook for the problems (the print version is out of date)

SIO 247: Lecture 3

- Math refresher (Appendix A.3)
- What is a magnetic field?
- Magnetic units
- Field as the gradient of a scalar potential
- A simple dynamo

- Scalars are 'just numbers'
- Vectors have length and direction





Python tricks

np.radians(angle_in_degrees)

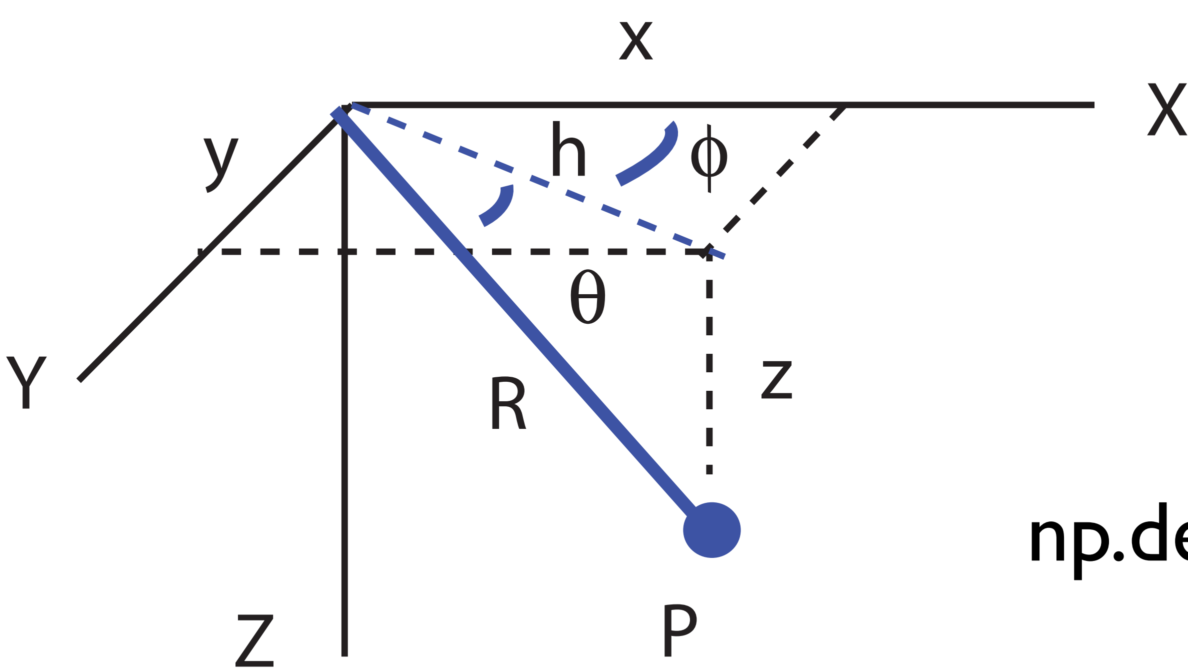
h=R*np.cos(theta)

$$h = R\cos\theta$$

$$x = h\cos\phi = R\cos\theta\cos\phi$$

$$y = h\sin\phi = R\cos\theta\sin\phi$$

$$z = R\sin\theta$$



Python tricks

np.degrees(phi_in_radians)

$$R = \sqrt{x^2 + y^2 + z^2}$$

np.sqrt(x**2+y**2+z**2)

$$\phi = \tan^{-1} \frac{y}{x}$$

np.arctan2(y,x)

$$\theta = \sin^{-1} \frac{z}{R}$$

np.arcsin(z/R)

Vector addition:

1) break each vector down into its components

$$A_x = |A| \cos\alpha, A_y = |A| \sin\alpha$$

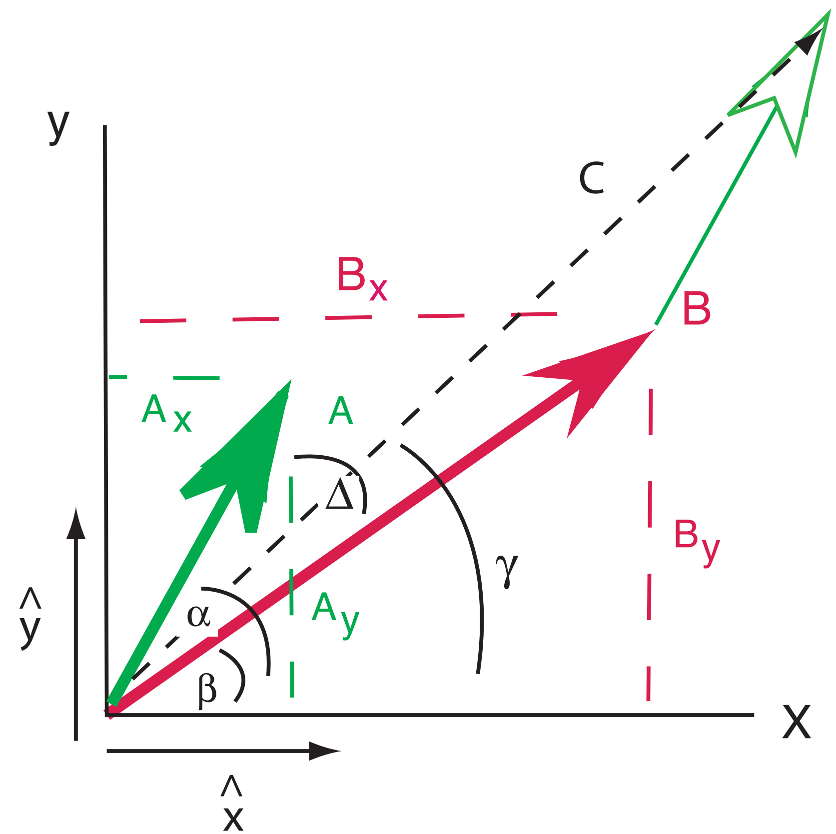
2) Add the components:

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

3) Convert back to:

|C| and γ



Addition and subtraction:

$\overrightarrow{A} + \overrightarrow{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z) = (A_x + B_x, A_y, +B_y, A_z + B_z)$

$\overrightarrow{A} - \overrightarrow{B} = (A_x, A_y, A_z) - (B_x, B_y, B_z) = (A_x - B_x, A_y, -B_y, A_z - B_z)$

Scalar multiplication:

$C \cdot \overrightarrow{A} = (C \cdot A_x, C \cdot A_y, C \cdot A_z)$

Dot product:

The dot product is the length of the projection of one vector over a second vector. Dot product is a scalar.

$\overrightarrow{A} \cdot \overrightarrow{B} = |\overrightarrow{A}||\overrightarrow{B}|cos\theta$, where θ is the angle between a and b. In cartesian coordinate system:

$\overrightarrow{A} \cdot \overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$

Python trick: np.dot(A,B) if A,B are arrays

Cross product:

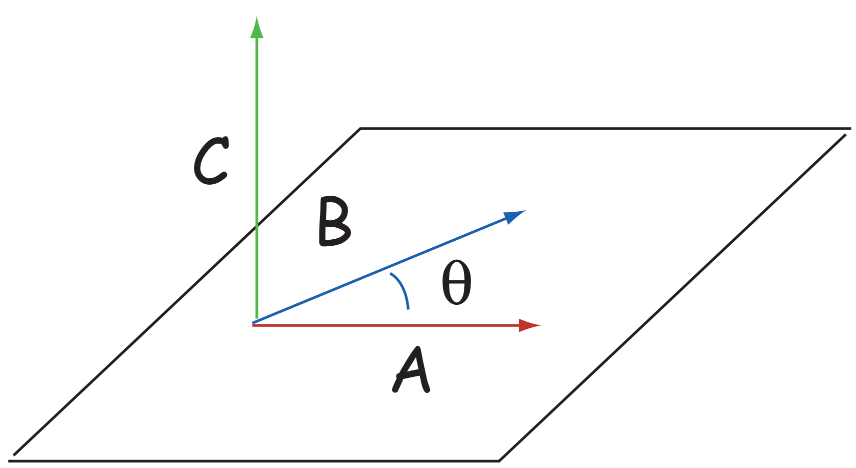
The cross product of A and B is a vector, which is perpendicular to both A and B.

In cartesian coordinate system:

$\vec{A} \times \vec{B} = ((A_y B_z - A_z B_y), (A_z B_x - A_x B_Z), (A_x B_y - A_y B_x)) = |\vec{A}||\vec{B}| \sin\theta$, where θ is the angle between a and b.

The cross operation can be written also in a form of a determinant:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



Python trick:

C=np.cross(A,B)

scalar field: $f = f(x, y, z)$

vector field: $F = \overrightarrow{F}(x, y, z)$

gradient of a scalar field: $grad(f) = \nabla(f)$

topography is a scalar field

direction and gradient of steepest descent is a vector field

Grad in cartesian coordinates:

$$\nabla(f) = (\hat{x}\frac{\partial}{\partial x}f + \hat{y}\frac{\partial}{\partial y}f + \hat{z}\frac{\partial}{\partial z}f)$$

Grad in spherical coordinates:

$$\nabla(f) = \frac{\partial f}{\partial r}\hat{r}, \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta}, \frac{1}{rsin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi}$$

SIO 247: Lecture 3

- Math refresher (Appendix A.3)
- What is a magnetic field?
- Magnetic units
- Field as the gradient of a scalar potential
- A simple dynamo

Start with Systeme International (SI) units

handy website for fundamental units:

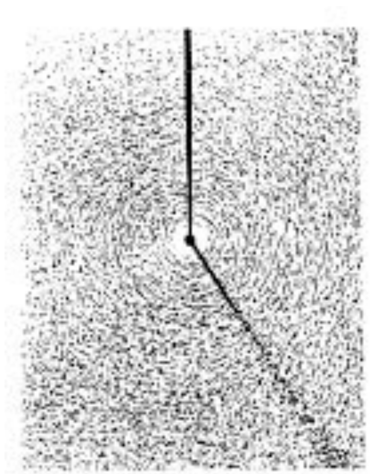
http://geophysics.ou.edu/solid_earth/notes/mag_basic/fundamental_units.htm

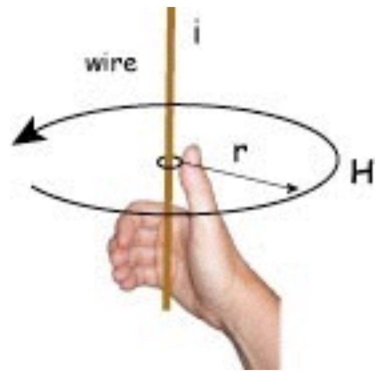
Quantity	Symbol	Derived Units	Dimensions
Energy, total	E	joule	ML^2T^{-2}
Energy, kinetic	K	joule	ML^2T^{-2}
Energy, potential	U	joule	ML^2T^{-2}
Force	**F**	newton	MLT^{-2}
Frequency	n	hertz, cycles/sec	T^{-1}
Gravitational field strength	**g**	nt/kg	LT^{-2}
Gravitational potential	V	joules/kg	L^2T^{-2}
Length	l	meter	L
Mass	m	kilogram	M
Mass density	r	kg/m^3	ML^{-3}
Momentum	**P**	$kg\text{-}m/sec^2$	MLT^{-1}
Period	T	second	T
Power	P	watt	ML^2T^{-3}
Pressure	p	nt/m^2	$ML^{-1}T^{-2}$

Magnetic units in SI:

Electric currents (i) make magnetic fields (H)

units for H:





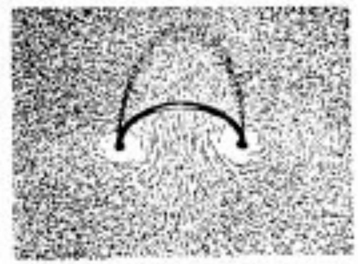
$$H = \frac{i}{2\pi r}$$

$$\frac{\text{Amperes}}{\text{meters}} \text{ or } \text{Am}^{-1}$$

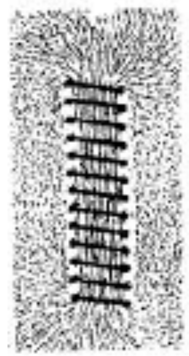
More general form of Ampere's Law: $\nabla \times \mathbf{H} = \mathbf{J}$

"The curl of H is the current density"

Let's bend the wire into a loop:



or several loops:



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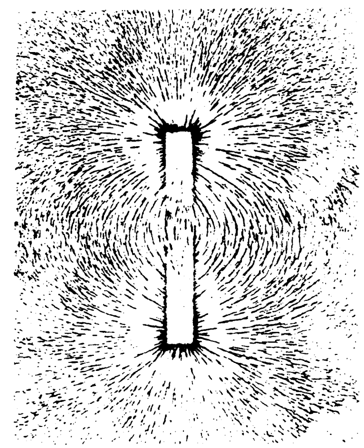

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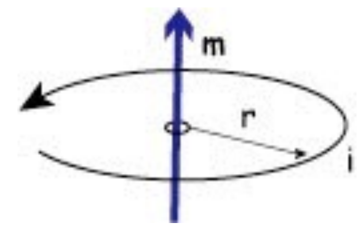


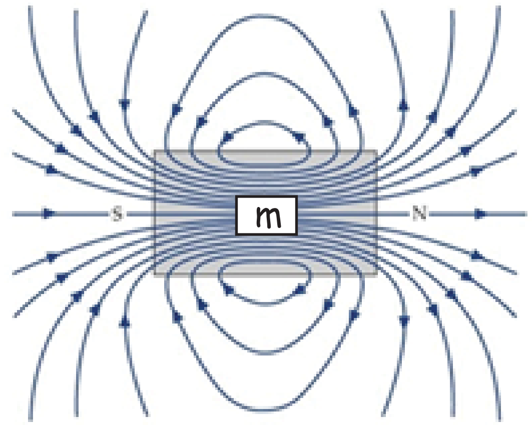
magnetic moment

$$m = ni\pi r^2$$

units for m: $\text{Amperes} \cdot \text{meters}^2 \text{or Am}^2$

$$\text{Magnetization} = \frac{\text{magnetic moment}}{\text{volume}} = \text{A m}^{-1}$$ (same as H)

Ways of thinking about magnetic fields



- magnetic fields are vector fields, having both direction and strength
- strength indicated by how close the lines are together (density of magnetic flux)
- direction indicated by directions of field lines (a.k.a. "lines of magnetic flux")

Could measure direction of H with a compass

But how to measure field strength?

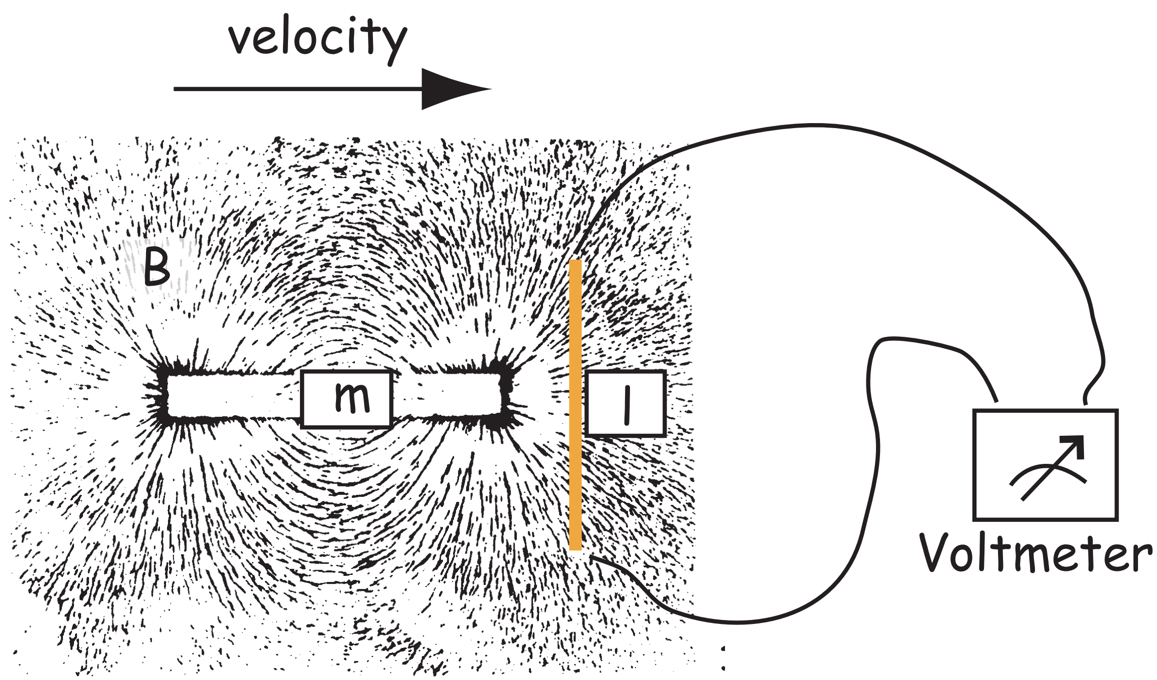
Moving charged particles (electric currents)
make magnetic fields (H)
and also
moving magnetic fields make electric currents

Let's call the magnetic field that induces the current
the "induction", B

B and H are obviously similar but they do NOT
(necessarily) HAVE THE SAME UNITS as we shall see

Could measure strength of the induction field like this:

move conductor of length *l* at velocity *v* through field *B* and generate potential *V*



Faraday's Law: $V = vlB$

$\text{Units of B: } \mathrm{V \cdot s \cdot m^{-2}}$

called a tesla (T)

But how are B and H related?

$$\mathbf{B} = \mu(\mathbf{H} + \mathbf{M})$$

μ is the "permeability"

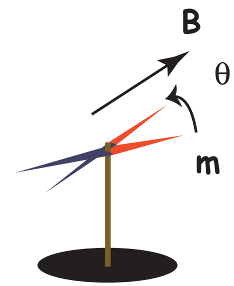
in free space M = 0 and

$\mu = \mu_o$, the permeability of free space

units of μ_o (in SI) are nasty!

working them out is a homework problem (1.4)

compasses move in response to the magnetic field



but what moves the magnet?

magnetic energy $$E_m = -\mathbf{m} \cdot \mathbf{B} = -mB\cos\theta$$

proof from dimensional analysis:

$$Am^2 \cdot \frac{kg}{As^2} = \text{joule}$$

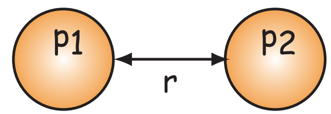
remember: $$E = [ML^2T^{-2}] = \text{joule}$$

Now let's do it in cgs

:)

Magnetic units in cgs:

Start with concept of a pair of magnetic monopoles with strength p_1, p_2



We make up a unit called "electrostatic units" (esu) for pole strength

Coulomb's Law for the force between two electric charges, q_1, q_2 is

$$F_{12} = k\frac{q_1 q_2}{r^2}$$

Adapting this to magnetic poles, we get: $F = \dfrac{p_1 p_2}{r^2}$

Force in cgs is dynes, so: $F = 1\mathrm{dyn} = \dfrac{1\mathrm{g\ cm}}{s^2} = \dfrac{1\ \mathrm{esu}^2}{cm^2}$

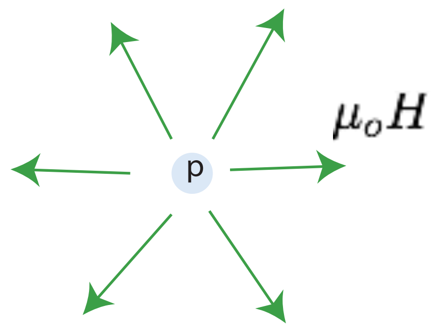
So the fundamental units of "esu" are: $1\mathrm{gm}^{1/2}\ \mathrm{cm}^{3/2}\ \mathrm{s}^{-1}$

Remember:

$$F = [MLT^{-2}] = \mathrm{newton\ (SI),\ dyne\ (cgs)}$$

What about the magnetic field (B,H)

The monopole creates a magnetic induction in the space around it

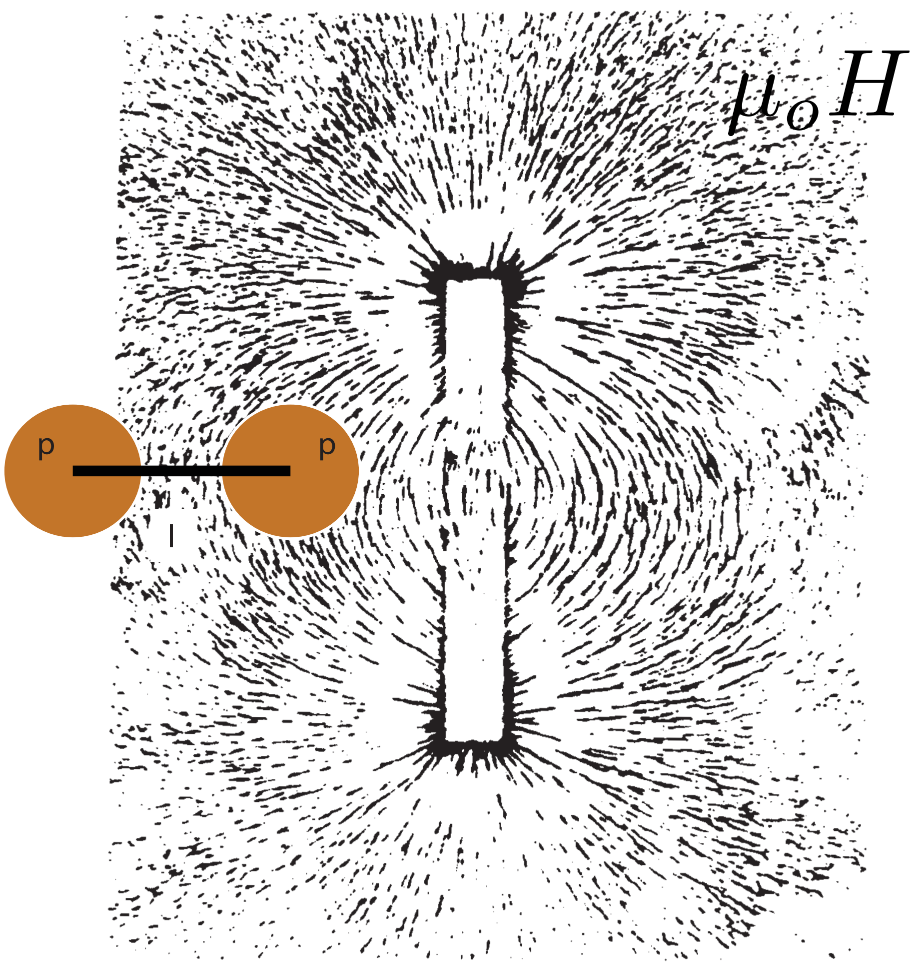


We make up a unit of field strength (H) called one oersted (Oe), defined as the field that exerts a force of 1 dyne on one unit of pole strength (an esu)

The related induction (B) $\mu_o H$ has units of gauss (G)

In cgs, the conversion between induction and field (μ_o) is unity

“What moves the magnet in cgs?



torque on magnet: $\Gamma = pl \times \mu_o \mathbf{H}$

Problem 1.2 is to write a python script to convert between the two systems

Table 1.1: Conversion between SI and cgs units.

Parameter	SI unit	cgs unit	Conversion
Magnetic moment (**m**)	Am^2	emu	1 A m^2 = 10^3 emu
Magnetization			
by volume (**M**)	Am^{-1}	emu cm^{-3}	1 Am^{-1} = 10^{-3} emu cm^{-3}
by mass (Ω)	Am^2kg^{-1}	emu gm^{-1}	1 Am^2kg^{-1} = 1 emu gm^{-1}
Magnetic Field (**H**)	Am^{-1}	Oersted (oe)	1 Am^{-1} = $4\pi \times 10^{-3}$ oe
Magnetic Induction (**B**)	T	Gauss (G)	1 T = 10^4 G
Permeability			
of free space (μ_o)	Hm^{-1}	1	$4\pi \times 10^{-7}$ Hm^{-1} = 1
Susceptibility			
total (K:**mH**)	m^3	emu oe^{-1}	1 m^3 = 10^6 4π emu oe^{-1}
by volume (χ: **M H**)	-	emu cm^{-3} oe^{-1}	1 S.I. = 1 _ 4π emu cm^{-3} oe^{-1}
by mass (κ:**m**$_m$ · 1 _ **H**)	m^3kg^{-1}	emu g^{-1} oe^{-1}	1 m^3kg^{-1} = 10^3 4π emu g^{-1} oe^{-1}

1 H = kg $m^2A^{-2}s^{-2}$, 1 emu = 1 G cm^3, $B = \mu_o H$ (in vacuum), 1 T = kg A^{-1} s^{-2}

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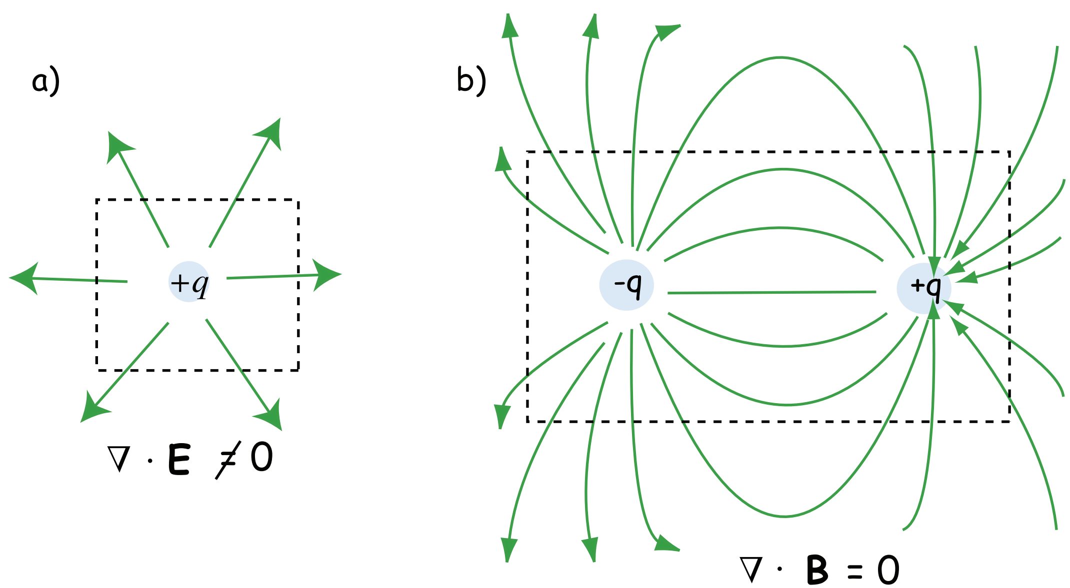
Unlike electrical fields (where electric charges produce fields that "diverge" away from the source) there are no isolated charges (monopoles) in nature

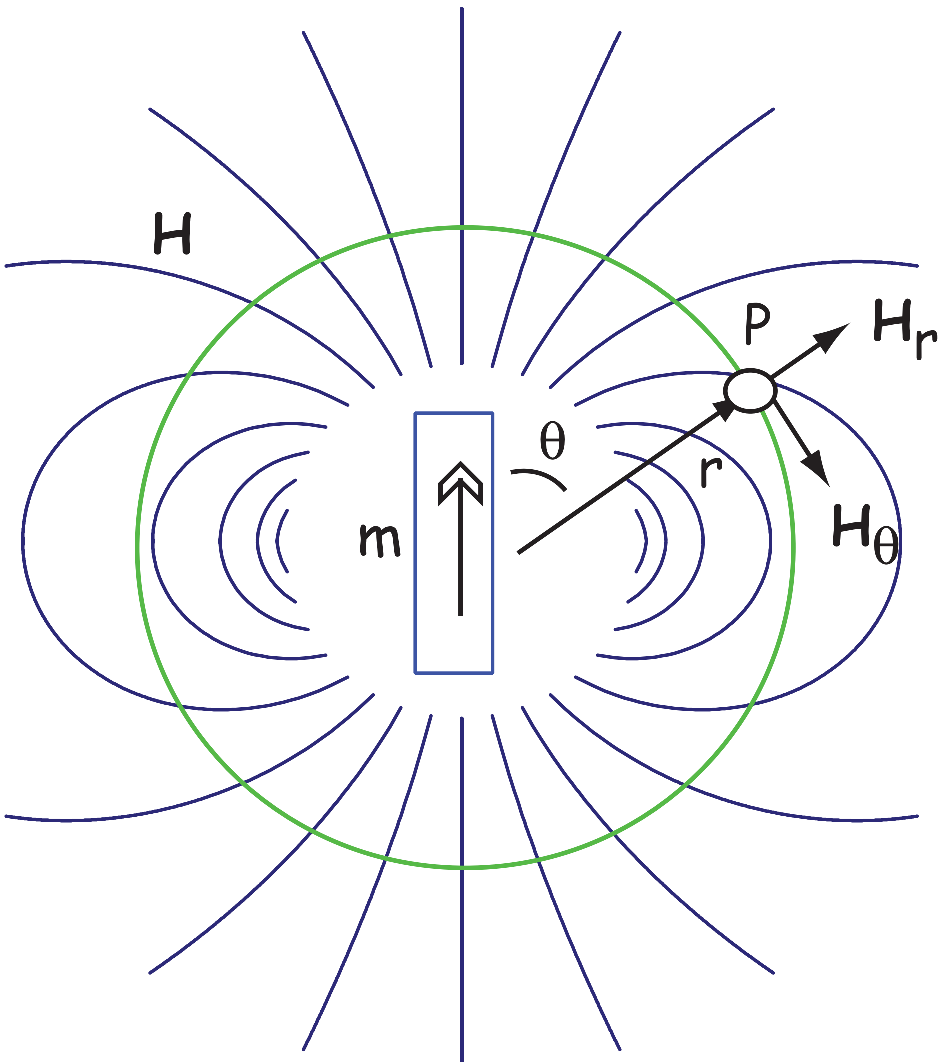


So the divergence of B is always zero AND, away from magnetic sources:

$$\mathbf{B} = \mu_o \mathbf{H}$$

In this special case can view magnetic field as gradient of a scalar potential:

$$\mathbf{H} = -\nabla\psi_m$$



$$\psi_m = \frac{\mathbf{m}\cdot\mathbf{r}}{4\pi r^3} = \frac{m\cos\theta}{4\pi r^2}$$

$$H_r = -\frac{\partial\psi_m}{\partial r} = \frac{1}{4\pi}\frac{2m\cos\theta}{r^3}$$

$$H_\theta = -\frac{1}{r}\frac{\partial\psi_m}{\partial\theta} = \frac{m\sin\theta}{4\pi r^3}$$

Problem 1.1: play with these equations

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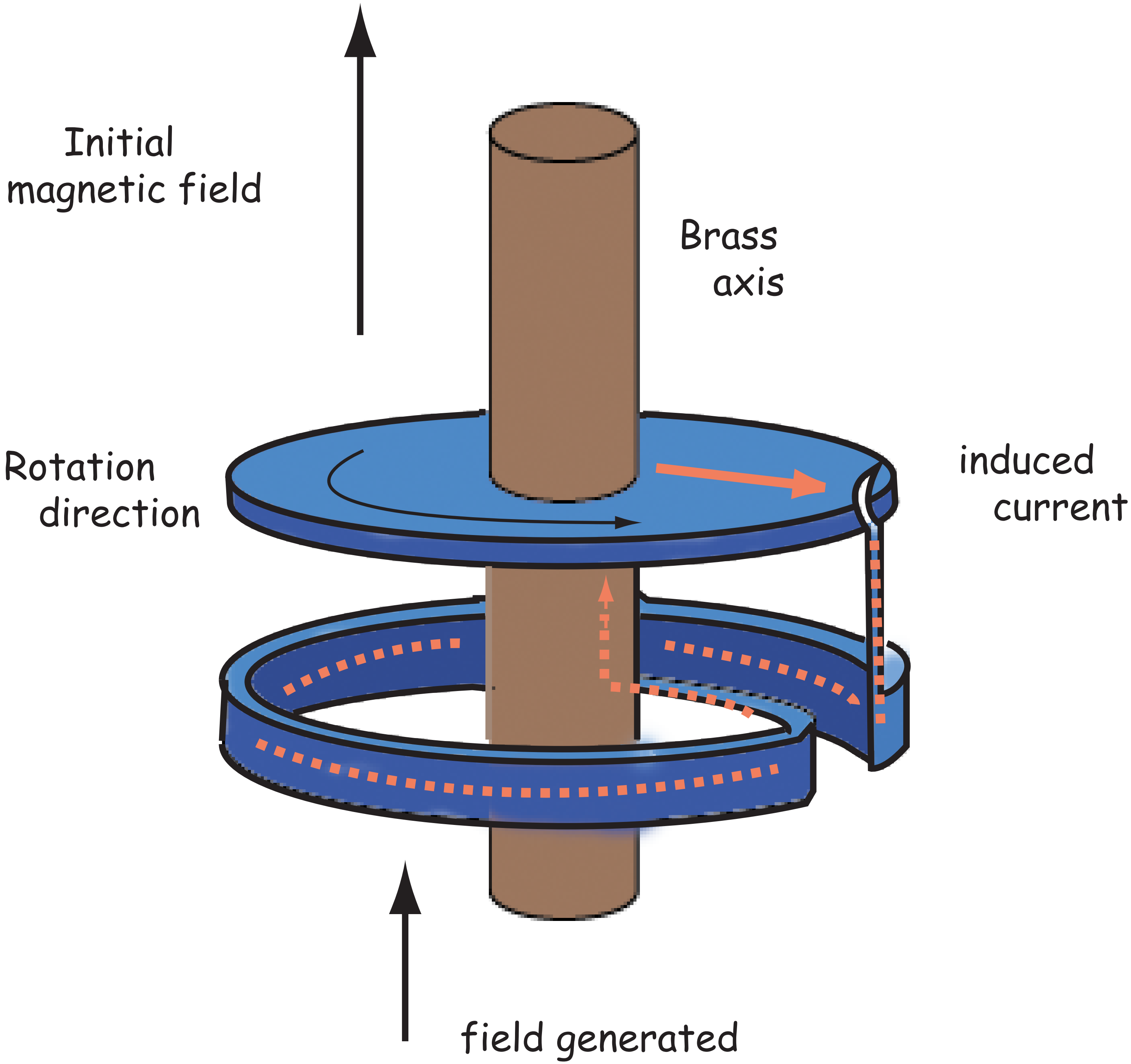
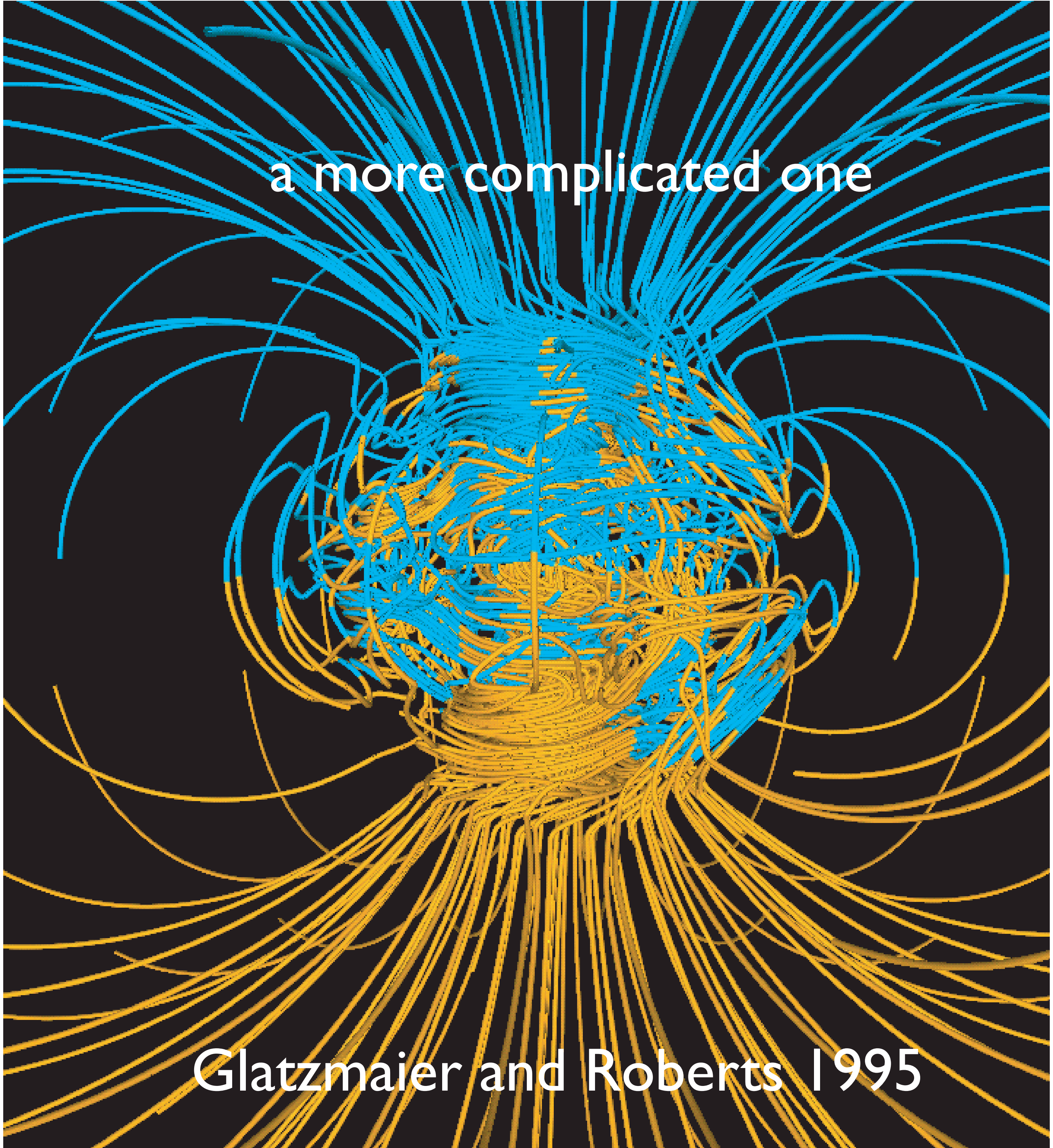


Initial
magnetic field
Brass
axis
Rotation
direction
induced
current
field generated


a more complicated one
Glatzmaier and Roberts 1995