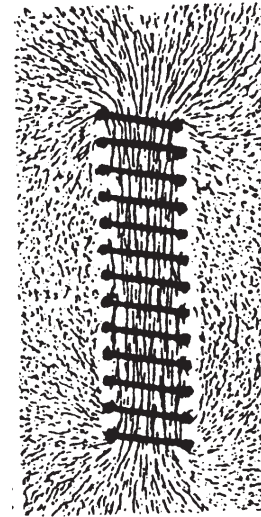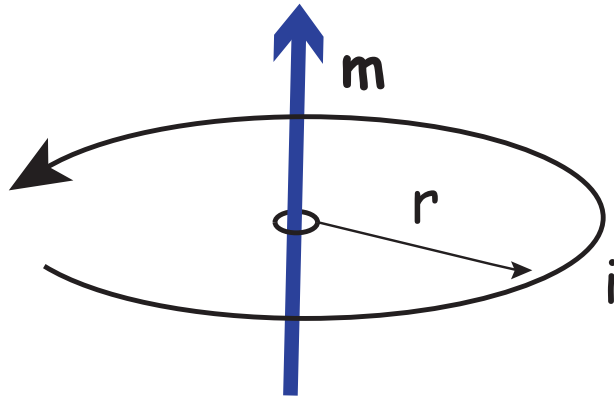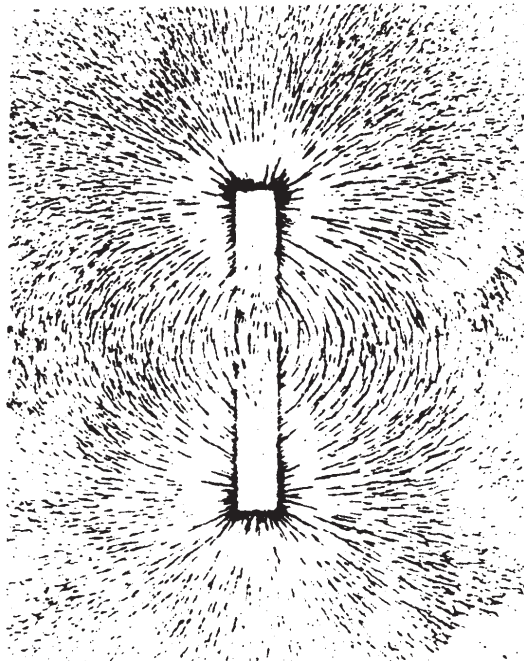# Lecture 5

- Induced magnetization: that which is induced in the presence of an applied magnetic field

    - diamagnetic

    - paramagnetic

- Remanent magnetization: that which remains in the absence of an external field
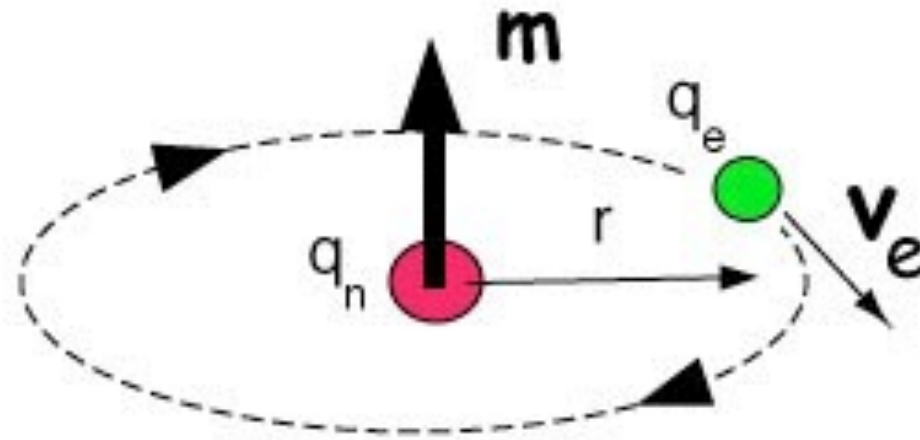
Chapters 3 & 4

From Chapter 1: "All magnetism results from currents"

So where are the currents here?

Classical view: electron scooting
around nucleus

Doesn't work because electron should eventually crash
into the nucleus and they don't

In quantum mechanics: orbit is stabilized.
Energy must be quantized
Electron must satisfy the "wave equation"

$$\Psi_{r,\theta,\phi} = R_{n,l}(r)Y_{l,m}(\phi,\theta)$$

r: radius

l: angular momentum

m: magnetic quantum number

Ψ^2 probability of finding
electron with r, θ, ϕ

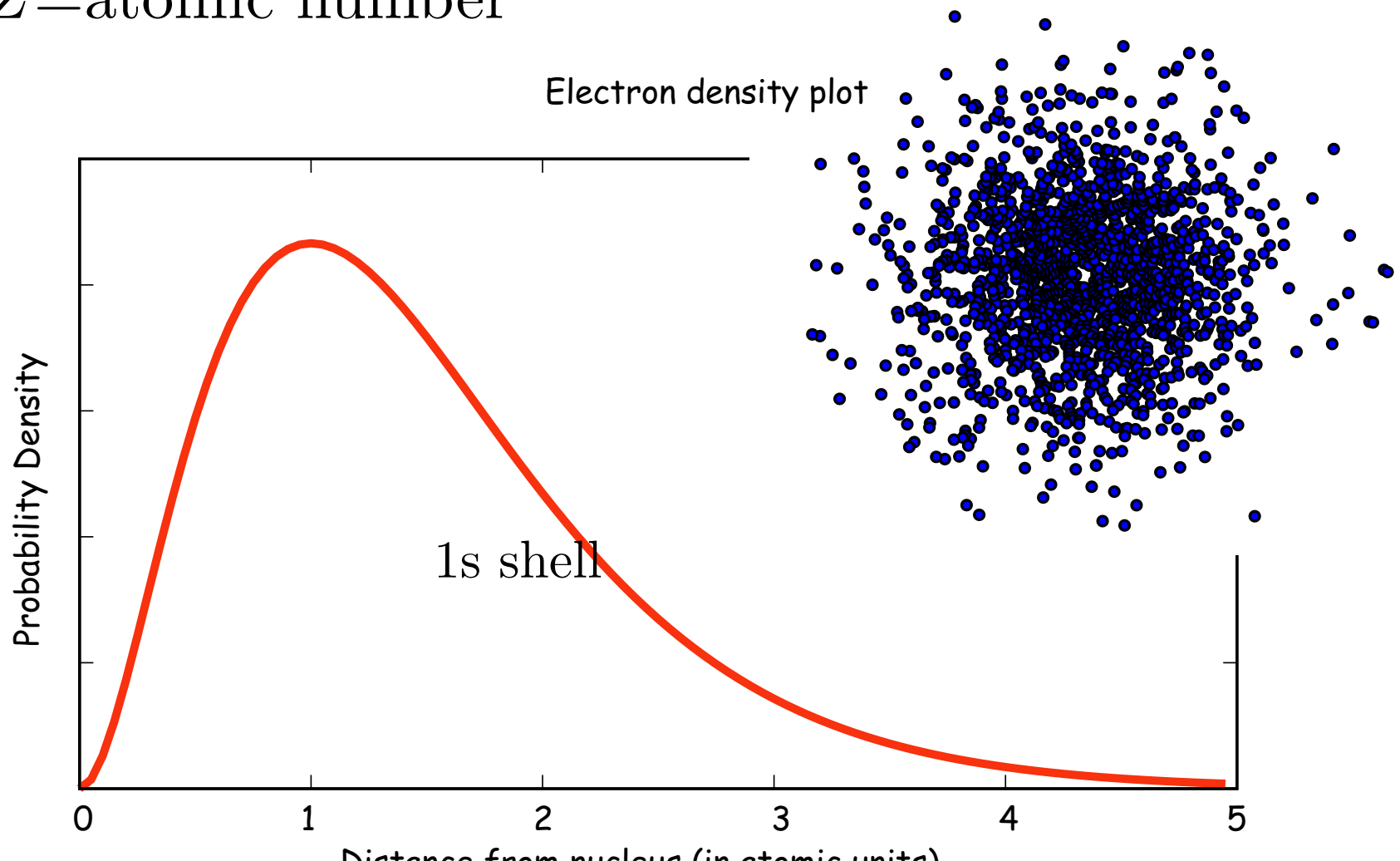$$\Psi_{r,\theta,\phi} = R_{n,l}(r)Y_{l,m}(\phi,\theta)$$

n,l,m are "quantum numbers"

- Principal (n=1,2,3,4): energy level of the shell
- Orbital angular momentum (l=0,1,… n-1): determines number of nodes (shape)
- Magnetic quantum number $(m_l = -l \rightarrow l$): number of orbitals and their orientation
- Electronic spin ($m_s = \pm\frac{1}{2}$): "up" or "down"

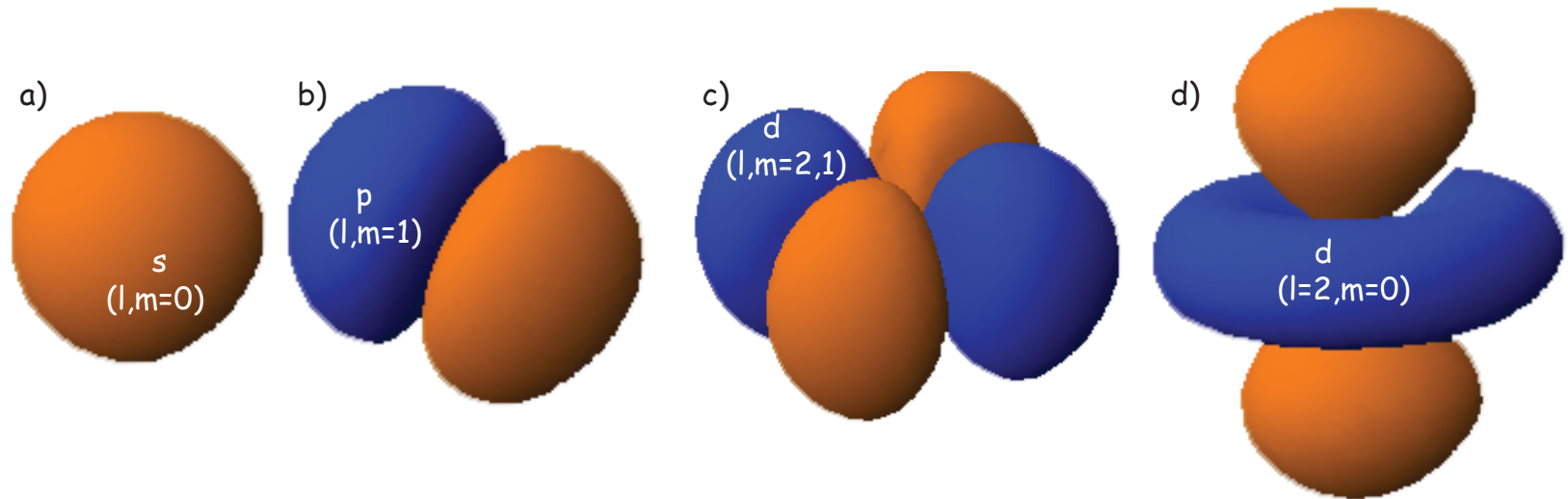$$\Psi_{r,\theta,\phi} = R_{n,l}(r)Y_{l,m}(\phi,\theta)$$

For   n=1, l,m=0   get lowest energy shell:

$$R_{1,0} = 2Z^{\frac{3}{2}}e^{-\frac{\rho}{2}}, Y_{0,0} = \left(\frac{1}{4\pi}\right)^{\frac{1}{2}}, \rho = 2Zr/n$$

and Z=atomic number

Electron density plot

1s shell

Probability Density

Distance from nucleus (in atomic units)

0   1   2   3   4   5

examples of energy shells:

a)
s
(l,m=0)

b)
p
(l,m=1)

c)
d
(l,m=2,1)

d)
d
(l=2,m=0)

For more examples checkout "Atom in a Box"

From Chapter 3:

one electron orbiting in one of these shells
generates a magnetic moment

$$m_b = \frac{\hbar q_e}{2\mu_e} = 9.2710^{-24} \frac{\text{kg m}^2}{\text{s}} \cdot \frac{\text{C}}{\text{kg}} = 9.2710^{-24} \text{Am}^2$$
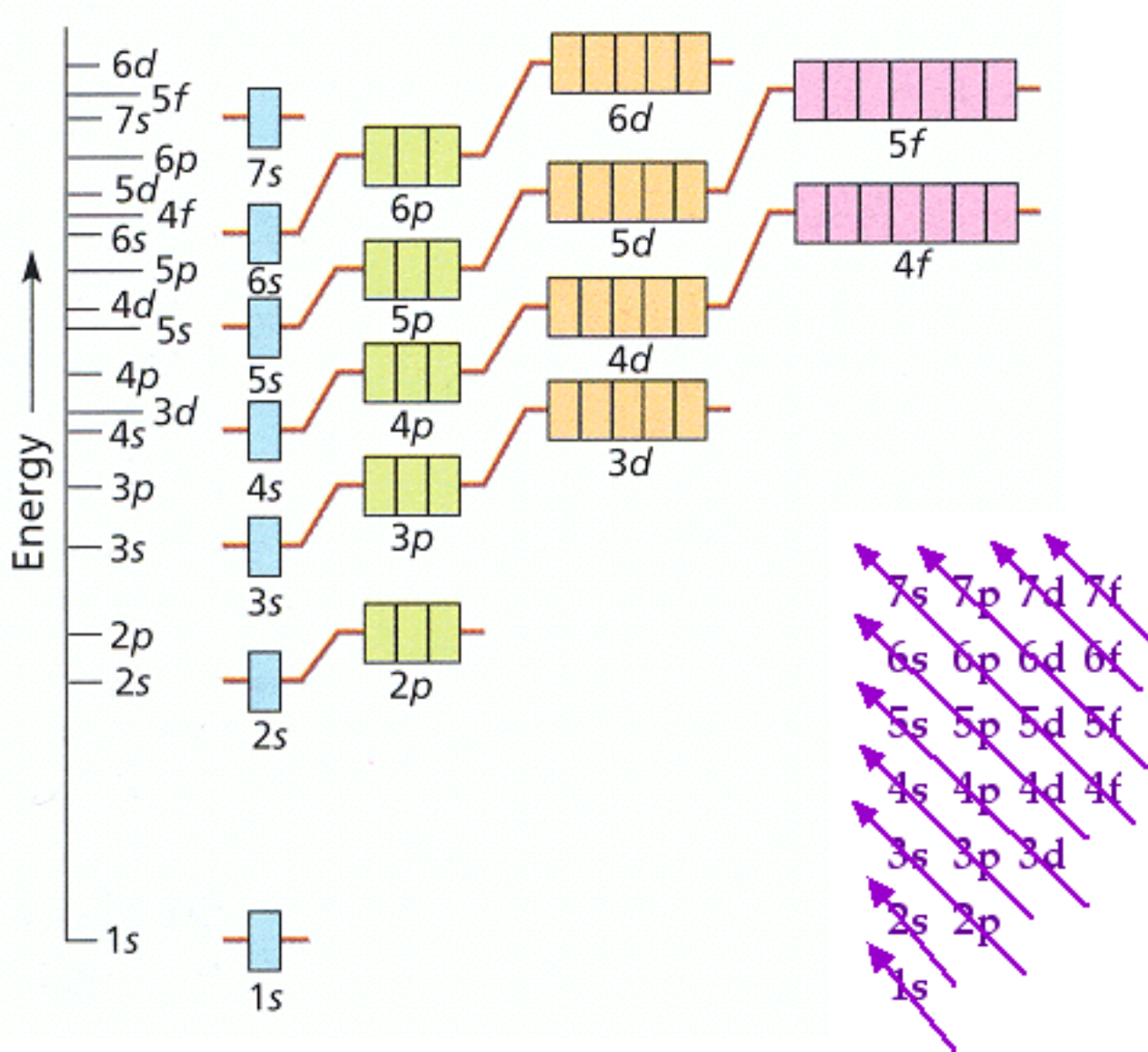
called one Bohr magneton - the smallest possible magnetic
moment

Note: angular momentum ($\hbar$): Js;   current: Amps = C/s;
mu_e is mass of one electron

Rules for filling electronic shells

- Hund's Rule: Electrons are added so that the spins remain as parallel as possible.

- Pauli's Exclusion principle : No two electrons may have the same set of quantum numbers. Because spin can be "up" or "down", two electrons fit in one shell

- Orbitals are filled in order of increasing energy.

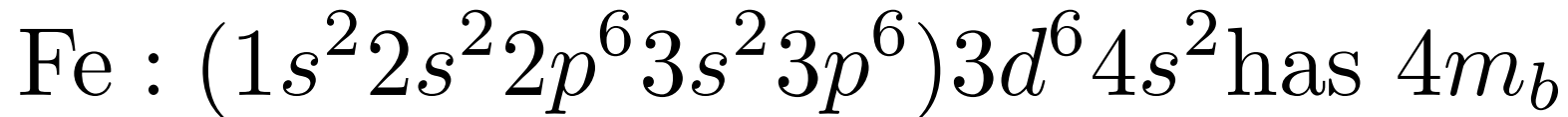# Orbitals filling up in order of increasing energy

look who has all the unpaired spins!

each unpaired spin also has a moment of one Bohr magneton

$$\text{Fe} : (1s^2 2s^2 2p^6 3s^2 3p^6) 3d^6 4s^2 \text{has } 4m_b$$

transition metals lose the 4s electrons first, so:

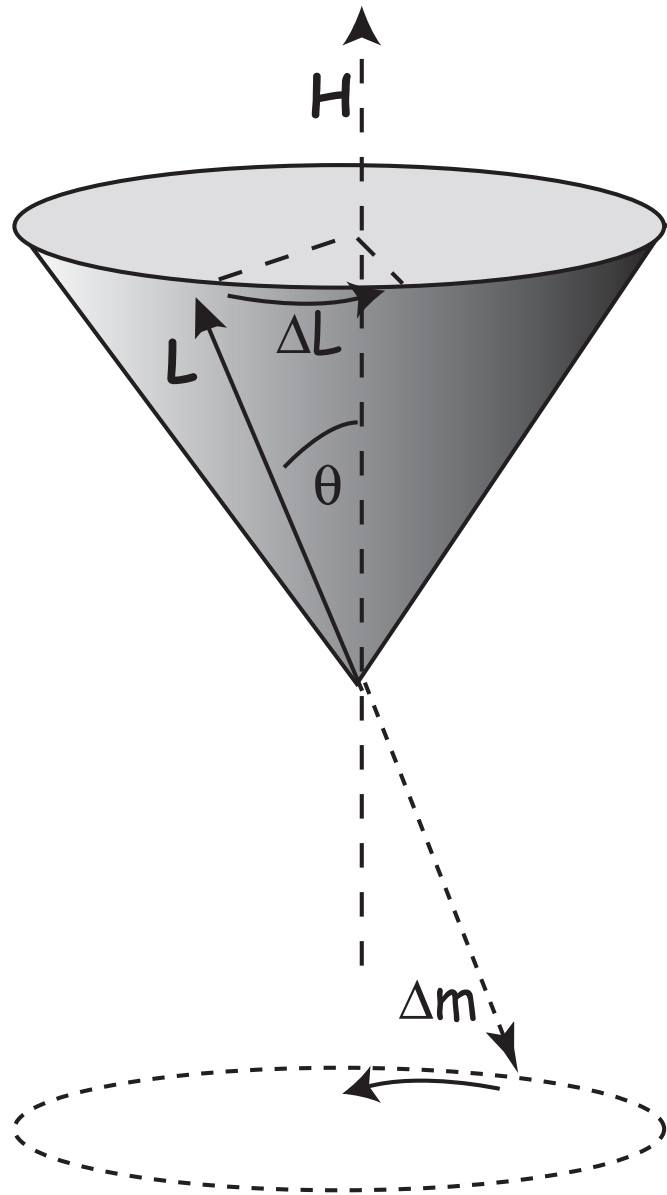$$\text{Fe}^{3+} : (1s^2 2s^2 2p^6 3s^2 3p^6) 3d^5 \text{has } 5m_b$$
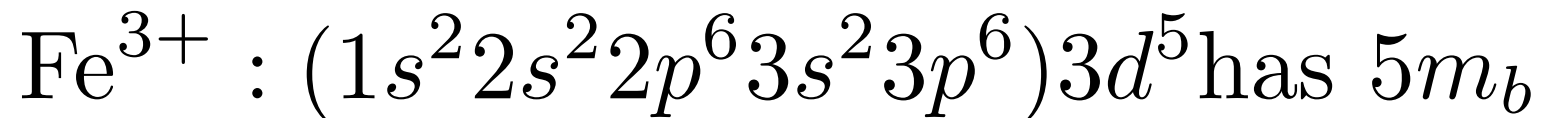
Induced magnetization:

comes from electronic orbits
AND
electronic spins

moment from electronic orbit

Electronic orbit has angular momentum vector $\mathbf{L}$

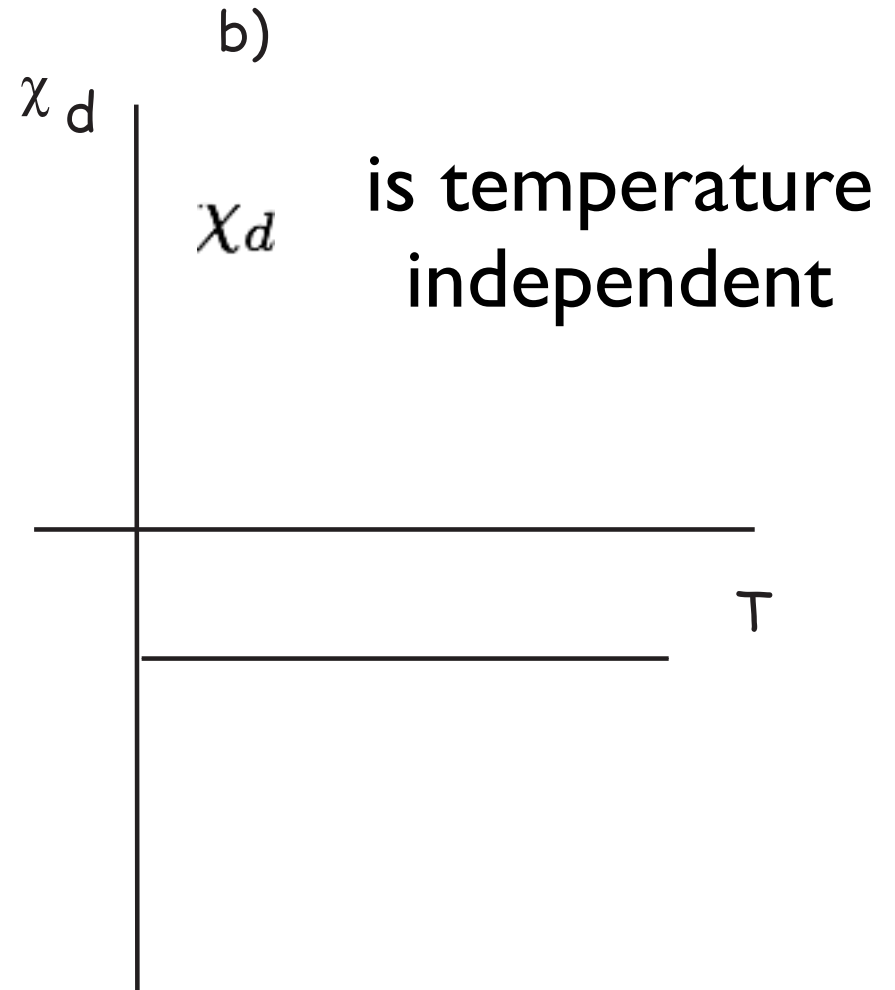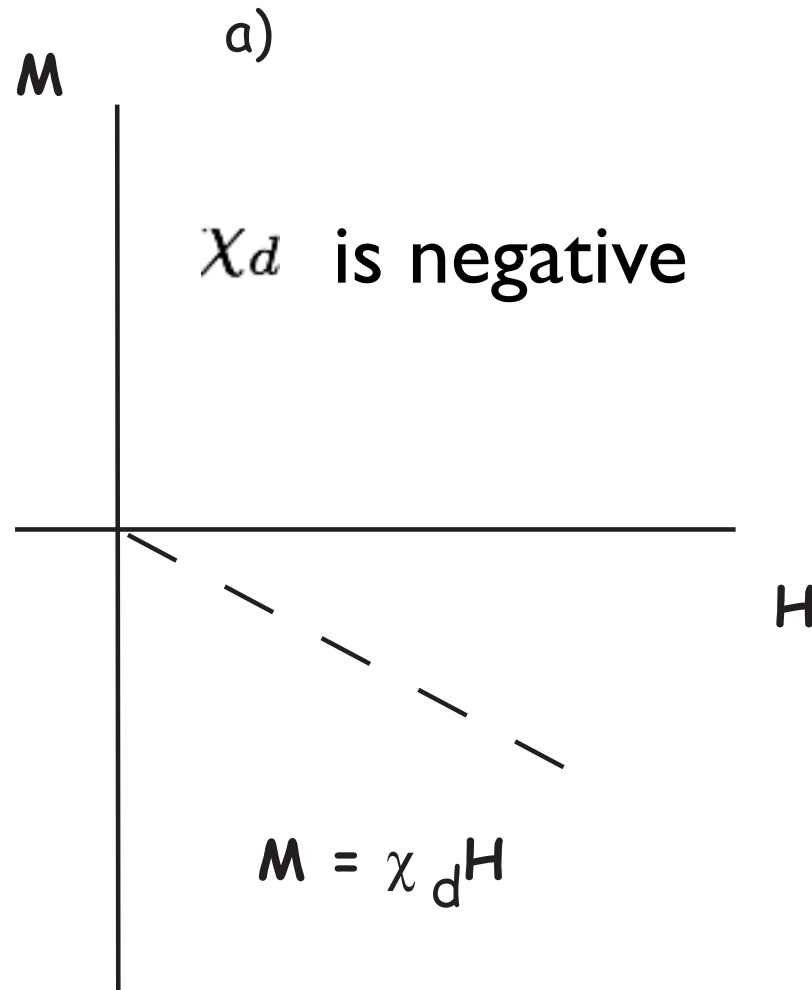H exerts a torque on L, shifting it by ΔL

(Larmor precession)

precession creates moment Δm

induced magnetization, $\mathbf{M}_I$ is volume normalized moment or A/m (same as H)

Orbitally induced magnetization called "diamagnetism"

$$\mathbf{M_I} = \chi_d \mathbf{H}$$

χ_d is diamagnetic susceptibility

a)

M

χ_d is negative

H

M = χ_dH

b)

χ_d

χ_d is temperature independent

T

Spin induced magnetization called "paramagnetism"

$$\mathbf{M}_I = \chi_p \mathbf{H}$$

χ_p is "paramagnetic susceptibility"

rules

- Each unpaired spin contributes a dipole moment (one Bohr magneton)

- In the absence of an applied field, the moments are essentially random

- An applied field acts to align the spins

- There is a competition between thermal energy (kT) and magnetic energy (mB cosθ)

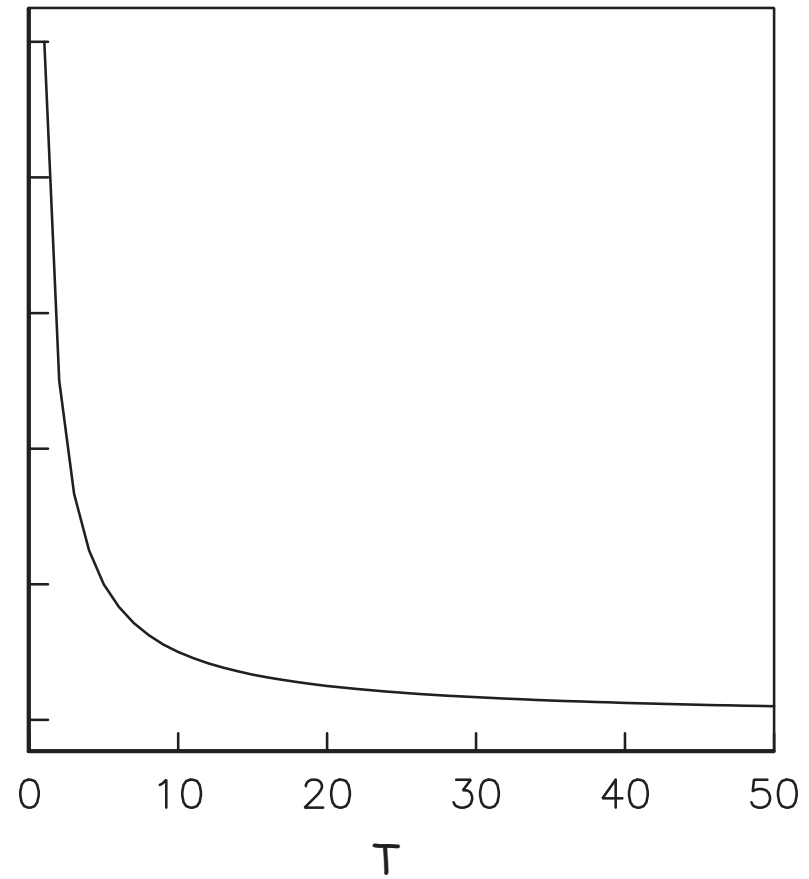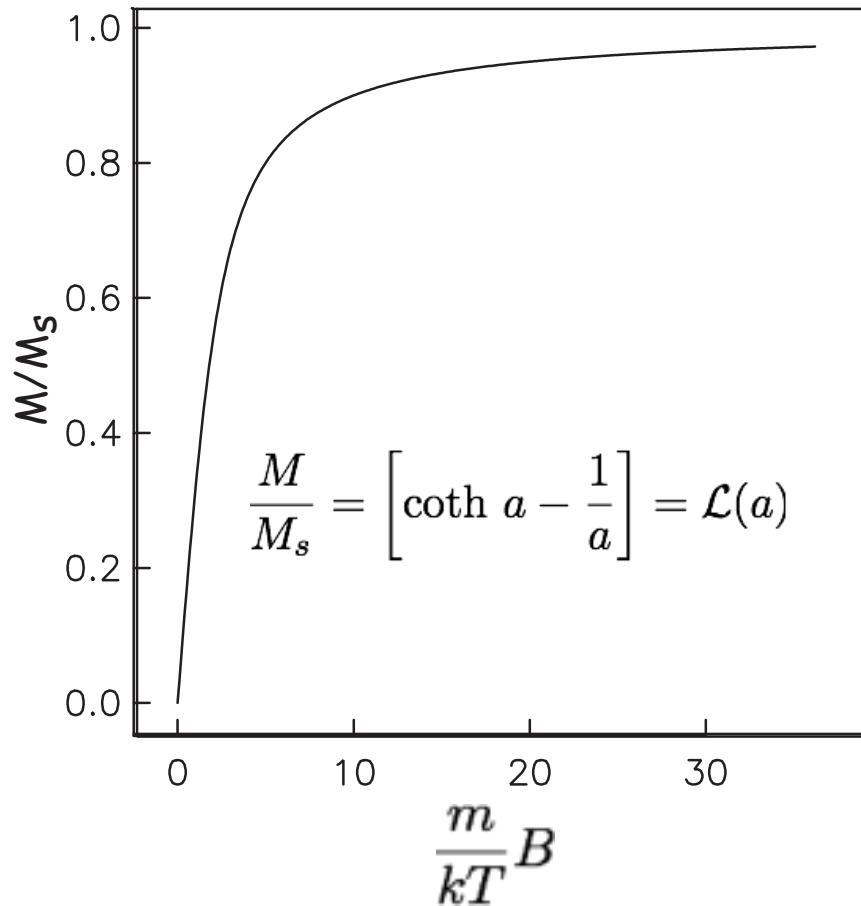Probability density of a given electron to have magnetic energy E_m

$$P(E) \propto \exp(-E_m/kT)$$

from this, magnetic energy, which is the degree of alignment with the field, depends on B and T

$$a = mB/kT$$

$$\frac{M}{M_s} = \left[\coth a - \frac{1}{a} \right] = \mathcal{L}(a)$$

(see Appendix A.2.1 for derivation)

$$\frac{M}{M_s} = \left[\coth a - \frac{1}{a}\right] = \mathcal{L}(a)$$

$$\frac{m}{kT}B = \chi_p B$$

χ_p is positive and has a strong temperature dependence

Also: a strong function of crystal alignment (anisotropy of magnetic susceptibility)

Magnetic remanence:

the magnetization that stays when the field turns off
a.k.a "spontaneous magnetization"
This is a property of CRYSTALS – not isolated atoms

- Because of Pauli's Exclusion principle (no two electrons with same set of quantum numbers), there is a tendency to avoid overlapping orbitals in neighboring electronic shells.

- Gives rise to an "exchange energy" in certain crystal structures. Equivalent to 1000 T (huge!)

- Exchange energy is minimized by aligning the spins either parallel or anti-parallel (depending on the nature of the interaction)

- Leads to cooperation of spins between neighboring electronic shells in certain crystals resulting in "spontaneous" magnetization (stays in absence of applied field)
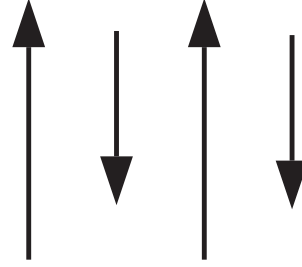
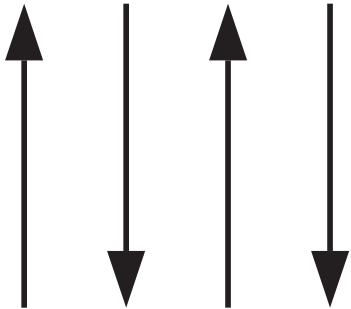and then there is "super-exchange": ties the spins of the Fe^{2+} to the Fe^{3+} across the O^{2-}

3d   2p   3d

Fe^{2+} O^{2-} Fe^{3+}
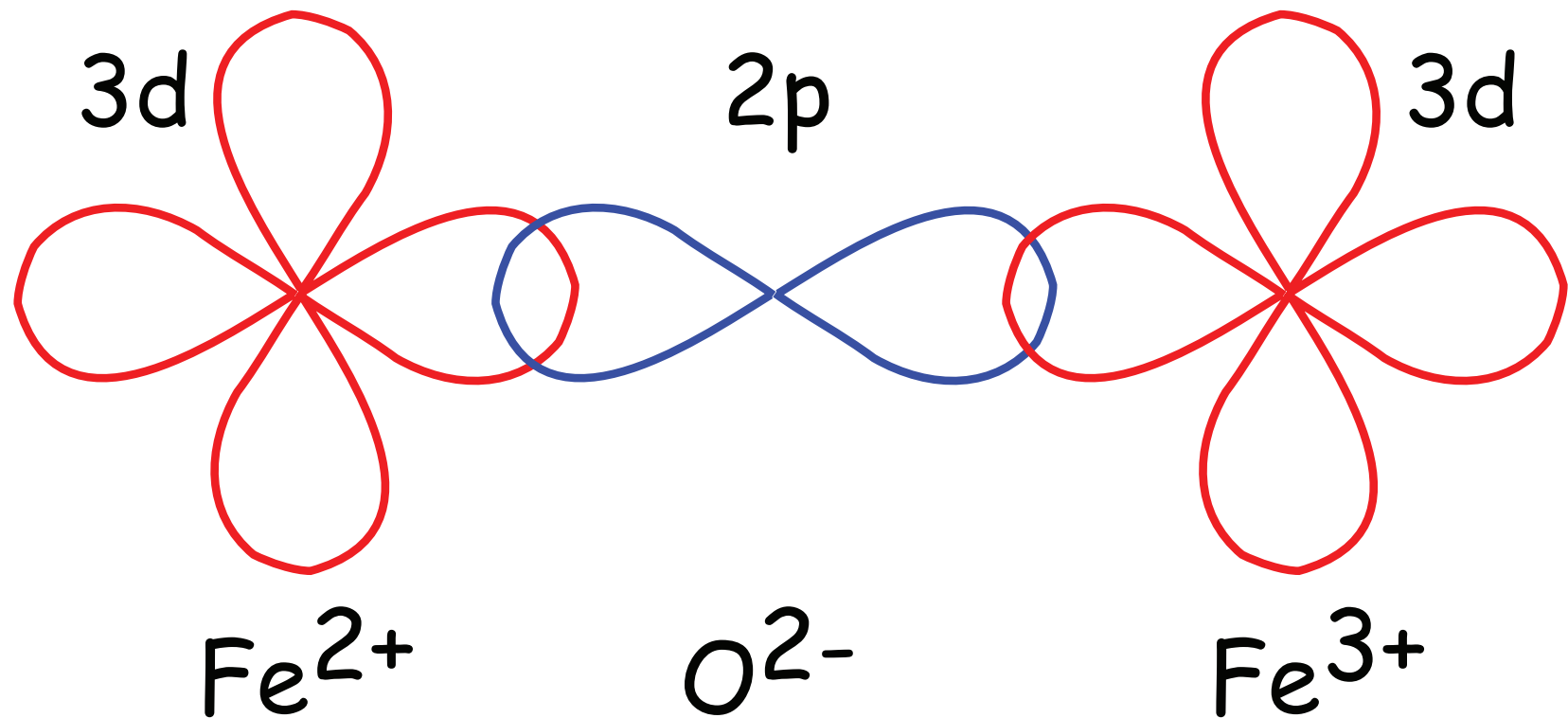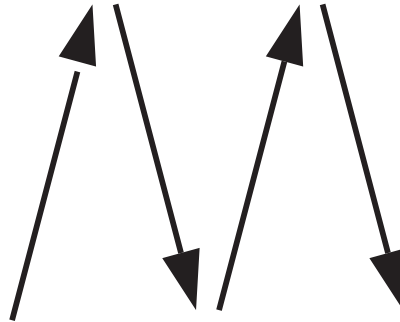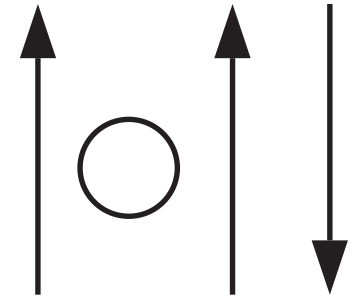
Types of spin alignment
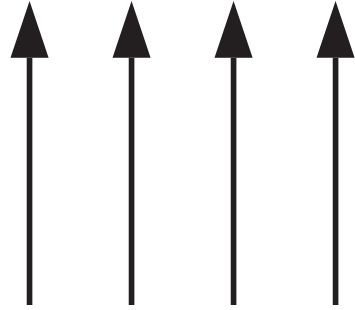
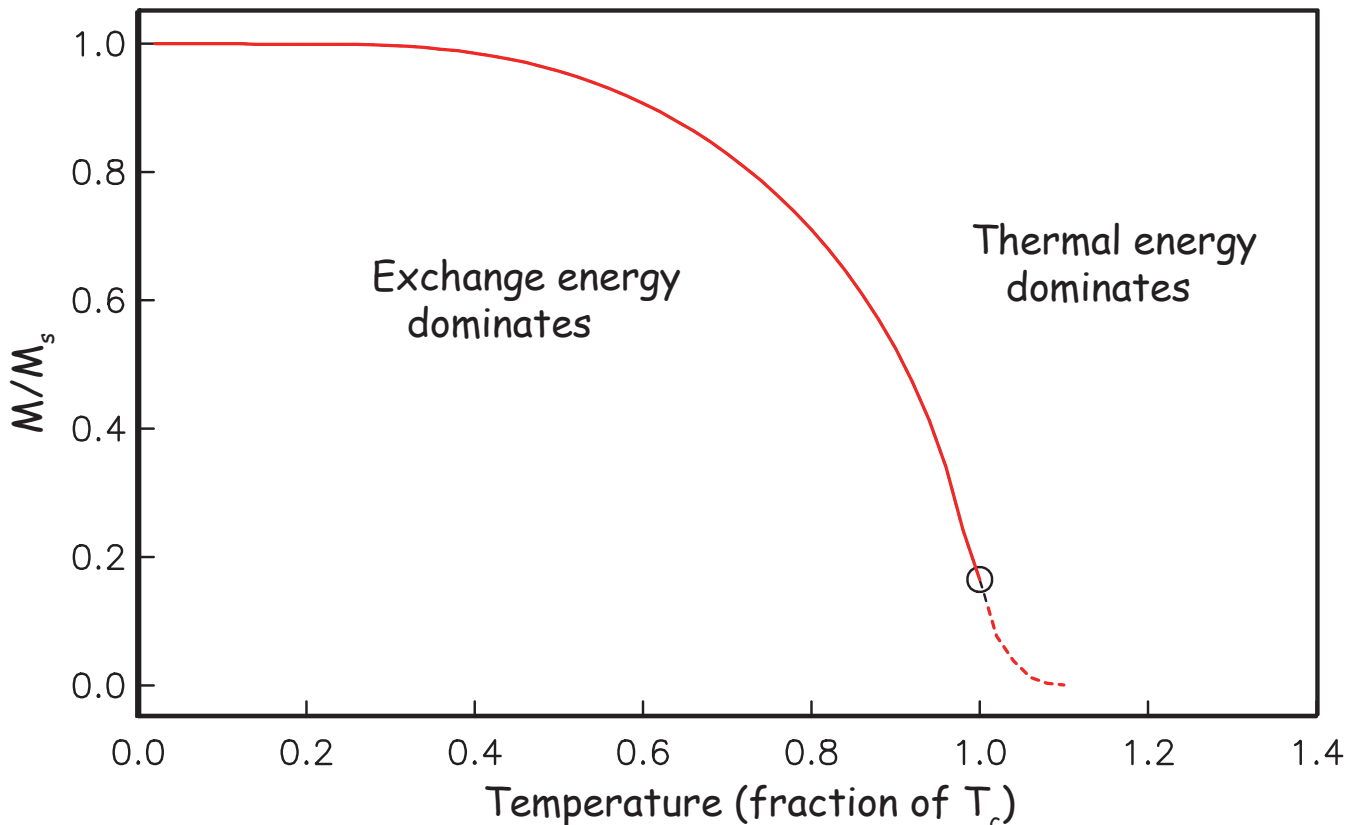**ferro**magnetic ferr**i**magnetic

anti-ferromagnetic spin canted anti-ferromagnetic defect

When all spins are perfectly parallel, magnetization is at saturation $M = M_s$

as T goes up, crystals expand and exchange energy becomes weaker - alignment (M) goes down

$$\frac{M}{M_s} < 1$$

$T > T_c$

exchange is zero and M is paramagnetic

So.. that was all about single
crystals - BUT

- Rocks contain assemblages of ferromagnetic (s.l.) minerals within matrix of diamagnetic/ paramagnetic minerals.

- For paleomagnetism, we need a frozen magnetization that is related to the geomagnetic field

Key questions:

- How does the average moment of an assemblage of particles come into equilibrium with an applied field (so that net M is proportional to B and the direction is parallel to B)?

- How does that net magnetization get frozen in so that we may measure it at some later time?

Key questions:

- How does the average moment of an assemblage of particles come into equilibrium with an applied field (so that net M is proportional to B and the direction is parallel to B)?

  - [Secret is "magnetic anisotropy energy" - certain directions within crystal are at lower energy than others]

- How does that net magnetization get frozen in so that we may measure it at some later time?

  - [Secret is that magnetic anisotropy energy can change from low, allowing magnetization to come into equilibrium with applied field to high, fixing the magnetization]

- So what IS magnetic anisotropy energy and how can it change?

Three sources of magnetic anisotropy energy

- Grain shape (magnetostatic)

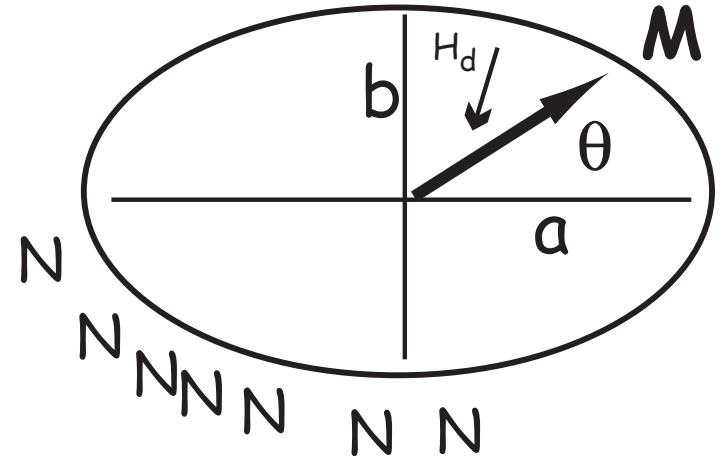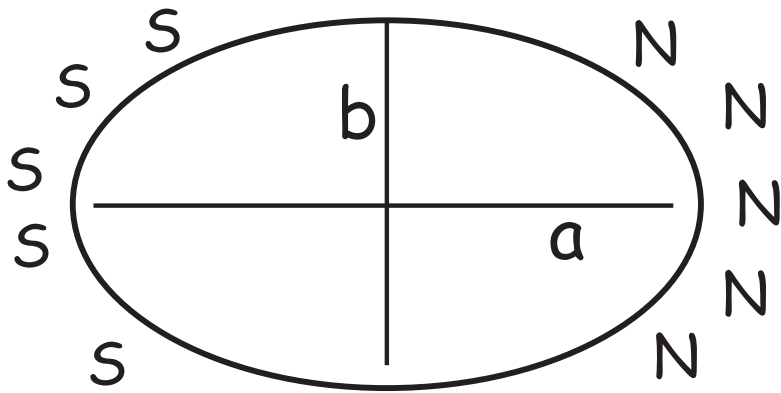- Crystal structure (magnetocrystalline)

- Strain (magnetostriction)

particle shape as a source of magnetic anisotropy energy
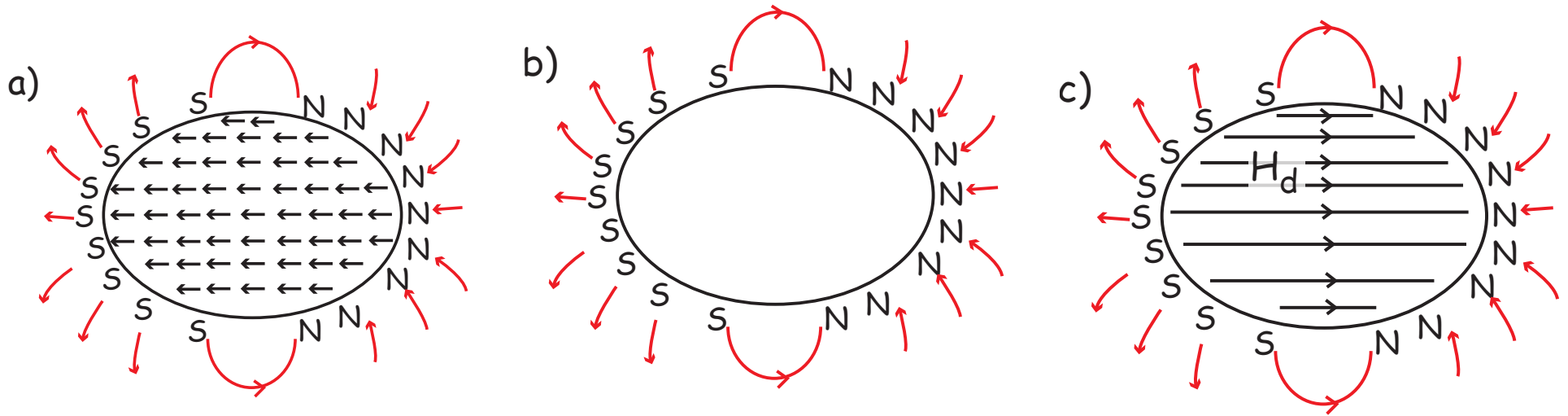
- demagnetizing field, Hd, is proportional to shape and to the magnetization:

$$\mathbf{H_d} = -\mathbf{NM}$$

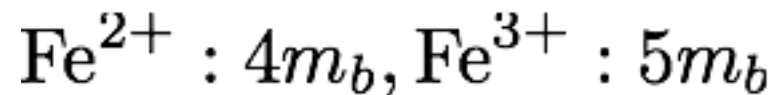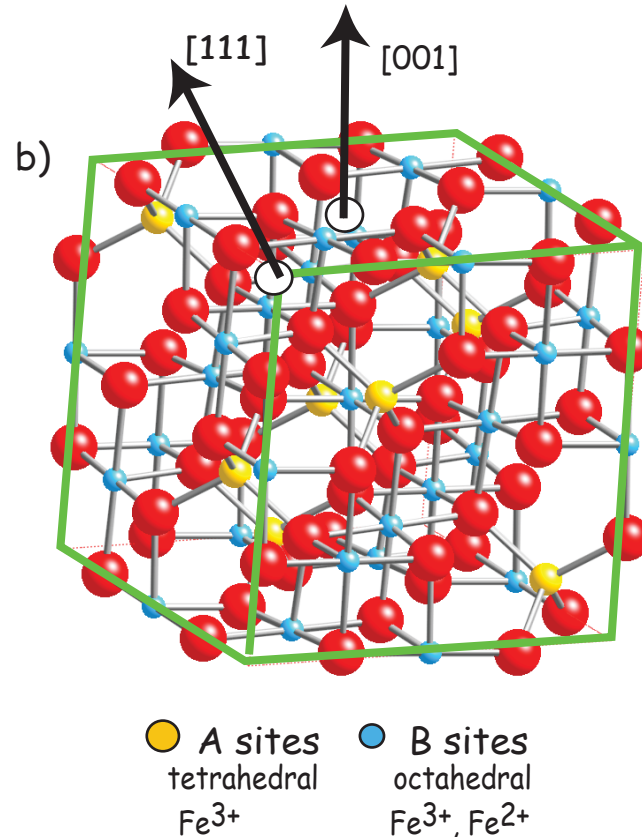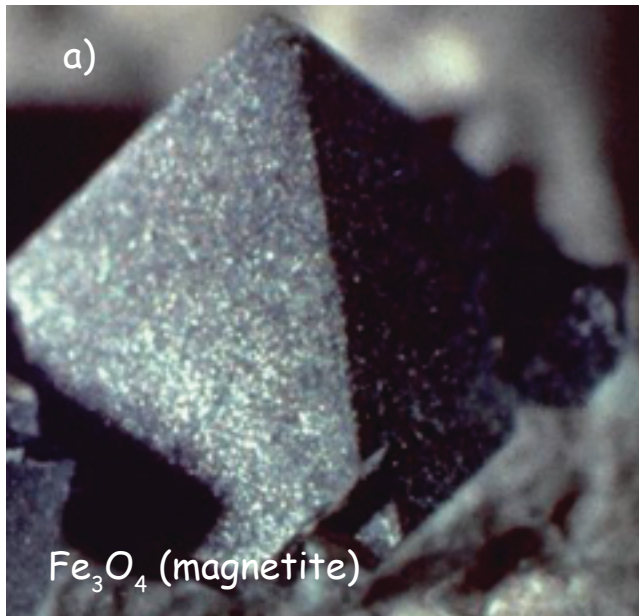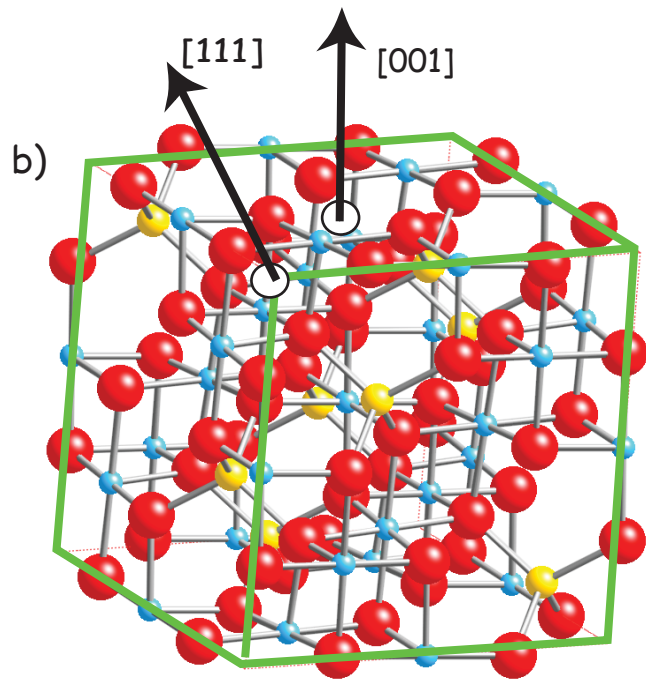- N is the "demagnetizing factor", a complicated tensor in the general sense.

- for prolate ellipsoid with demagnetizing factors Na and Nb parallel to a and b axes

- boils down to $\epsilon_d = -\frac{1}{2}\mu_o(N_b \sin^2 \theta)M^2$

- minimized when M is parallel to long axis

- uniaxial anisotropy energy density constant:

$$K_u = \frac{1}{2}\Delta N \mu_o M^2$$

Crystal structure

a) Fe$_3$O$_4$ (magnetite)

b) [111] [001]

A sites
tetrahedral
Fe^{3+}

B sites
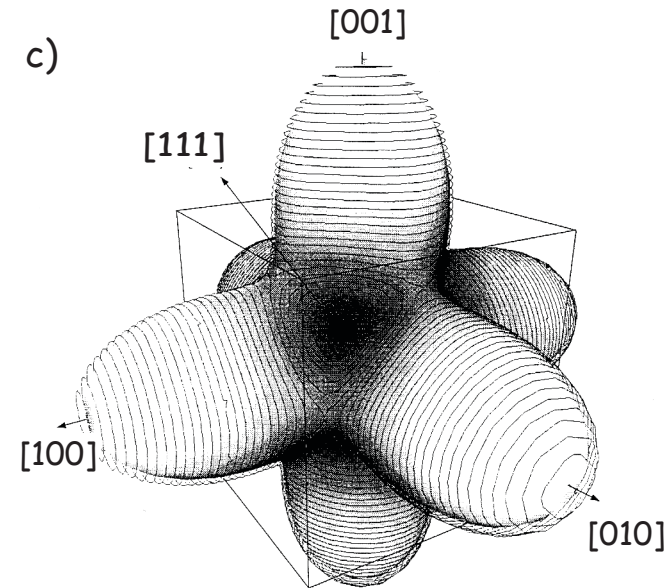octahedral
Fe^{3+}, Fe^{2+}

$$Fe^{2+} : 4m_b, Fe^{3+} : 5m_b$$

exchange energy aligns all A site ions (5 mb) parallel to each other and all B site ions (9 mb) anti-parallel to A; net magnetization of 4 mb

b)

[111] [001]

A sites
tetrahedral
Fe^{3+}

B sites
octahedral
Fe^{3+}, Fe^{2+}

c)

[001]

[111]

[100]

[010]

The magnetic energy of the crystal is a function of orientation of the spins
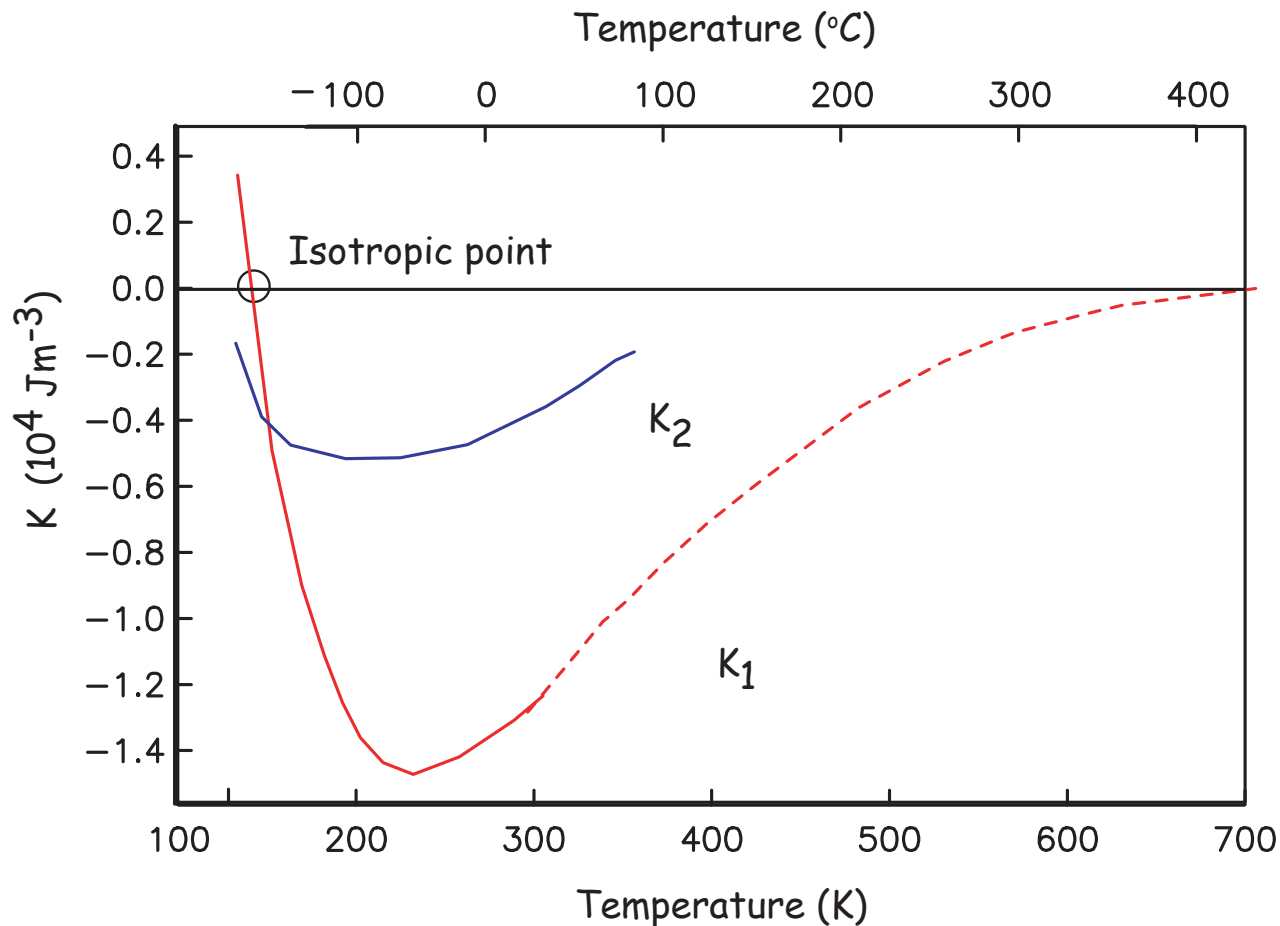
called magnetocrystalline anisotropy energy

certain directions (e.g., parallel to 111 in magnetite) have lower energies than others (e.g. 001)

fixes moment in "easy" directions

magnetocrystalline anisotropy energy density as a function of direction and T

$$\epsilon_a = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + K_2\alpha_1^2\alpha_2^2\alpha_3^2,$$

magnetic anisotropy energy due to stress

- because magnetization results from interaction of electronic orbitals, straining a crystal will result in changes in the magnetization

- in turn, the magnetization itself will change the shape of the crystal (magnetostriction)

- there is an energy associated with this (see book)

Key points from lecture

- magnetization in substances arises from electronic orbits (diamagnetism), spins (paramagnetism) and cooperative spin behavior in crystals (ferromagnetism)

- the key to paleomagnetism is first allowing magnetizations to come into equilibrium with external field, then freezing them in place

- secret is magnetic anisotropy energy

Assignment

Problems 2.3 and 3.1 in 4th Edition online book